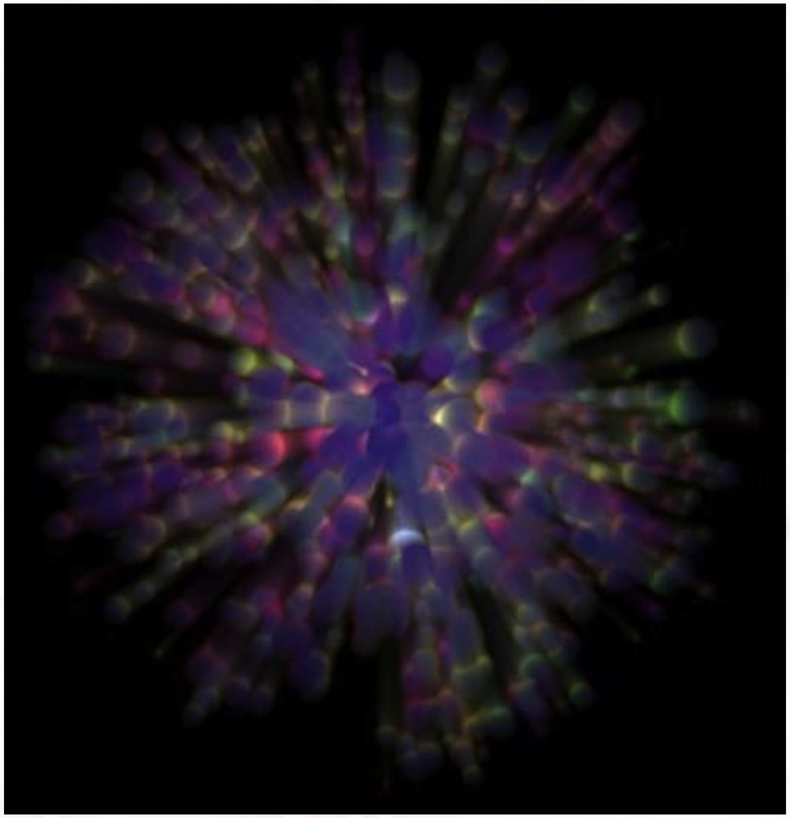
Implications of dark matter, electron screening and statistics on big bang nucleosynthesis



C.A. Bertulani

Department of Physics and Astronomy
Texas A&M University-Commerce

*E-mail: **carlos.bertulani@tamuc.edu***

*URL: **http://faculty.tamuc.edu/cbertulani/***

Standard BBN

a: scale factor

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\left(\rho_\gamma + \rho_{e^\pm} + \rho_b + \rho_\nu\right)} \equiv H$$

Friedman Eq.

ρ: energy density of relativistic species (m < 1 MeV)

$$\dot{\rho} = -3H(\rho + p)$$

μ_e: electron chemical potential

$$n_b \sum_j Z_j X_j = n_{e^-} - n_{e^+} \equiv \Phi\left(\frac{m_e}{T}, \mu_e\right)$$

$$\dot{X}_i = \sum_{j,k,l} N_i \left(\Gamma_{kl\to ij} \frac{(X_l)^{N_l}}{N_l!} \frac{(X_k)^{N_k}}{N_k!} - \Gamma_{ij\to kl} \frac{(X_i)^{N_i}}{N_i!} \frac{(X_j)^{N_j}}{N_j!} \right) \equiv \Gamma_i$$

BBN - Successes

As long as the weak reactions are fast enough, the neutron-to-proton ratio is given by

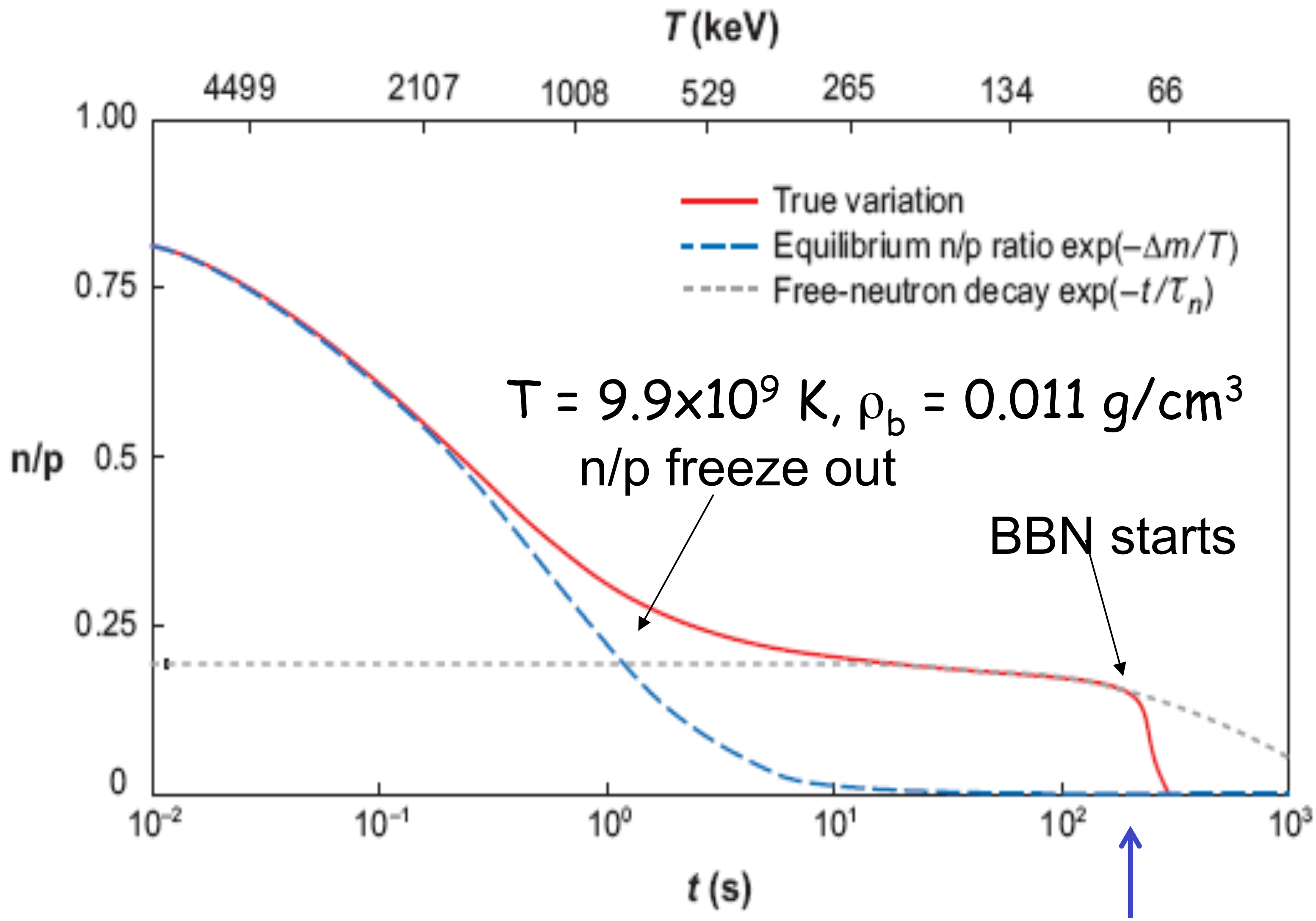
$$[n/p] = \frac{\text{number of neutrons}}{\text{number of protons}} = \frac{N_n(T)}{N_p(T)} = \exp\left[-\frac{\Delta mc^2}{kT}\right]$$

(mc^2 = 1.29 MeV).

At the freeze-out temperature T ~ 7 × 10^9 K (about 700 keV), [n/p] ~ 1/6.

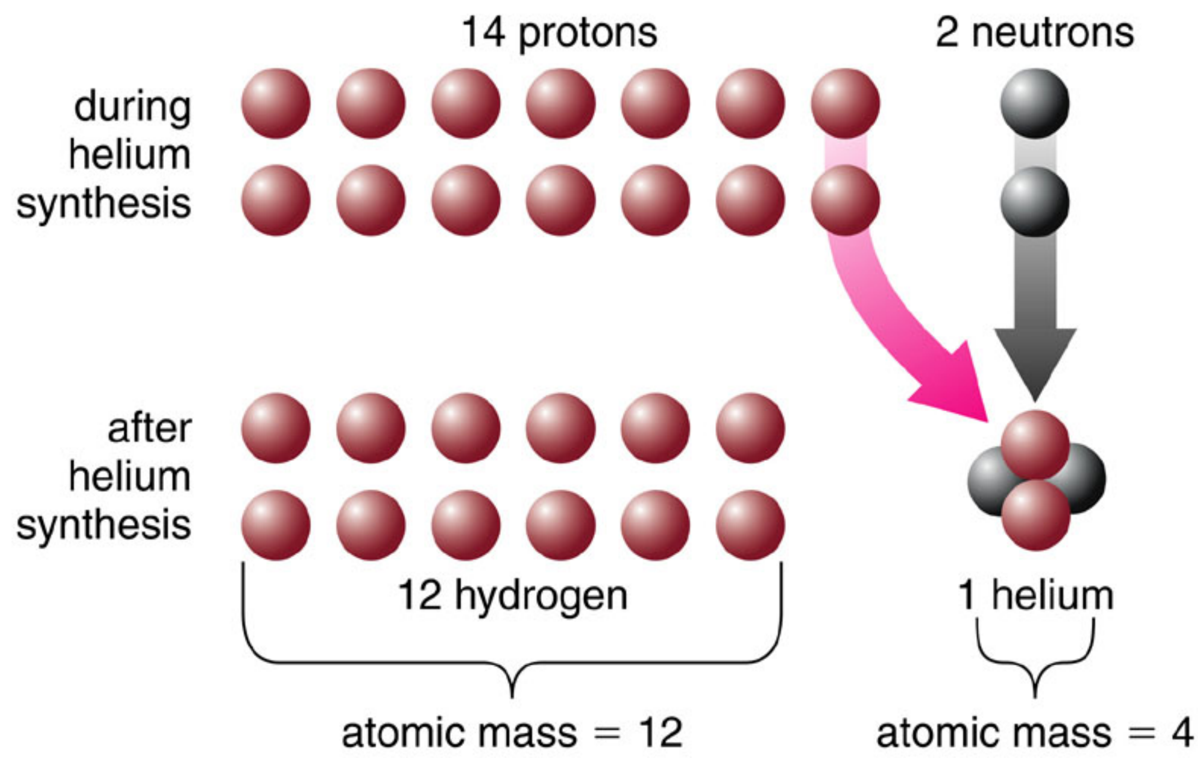
BBN

G. Steigman, ARNPS 57, 463 (2007)



$t_{BBN} \approx 4 \rightarrow 24$ min.

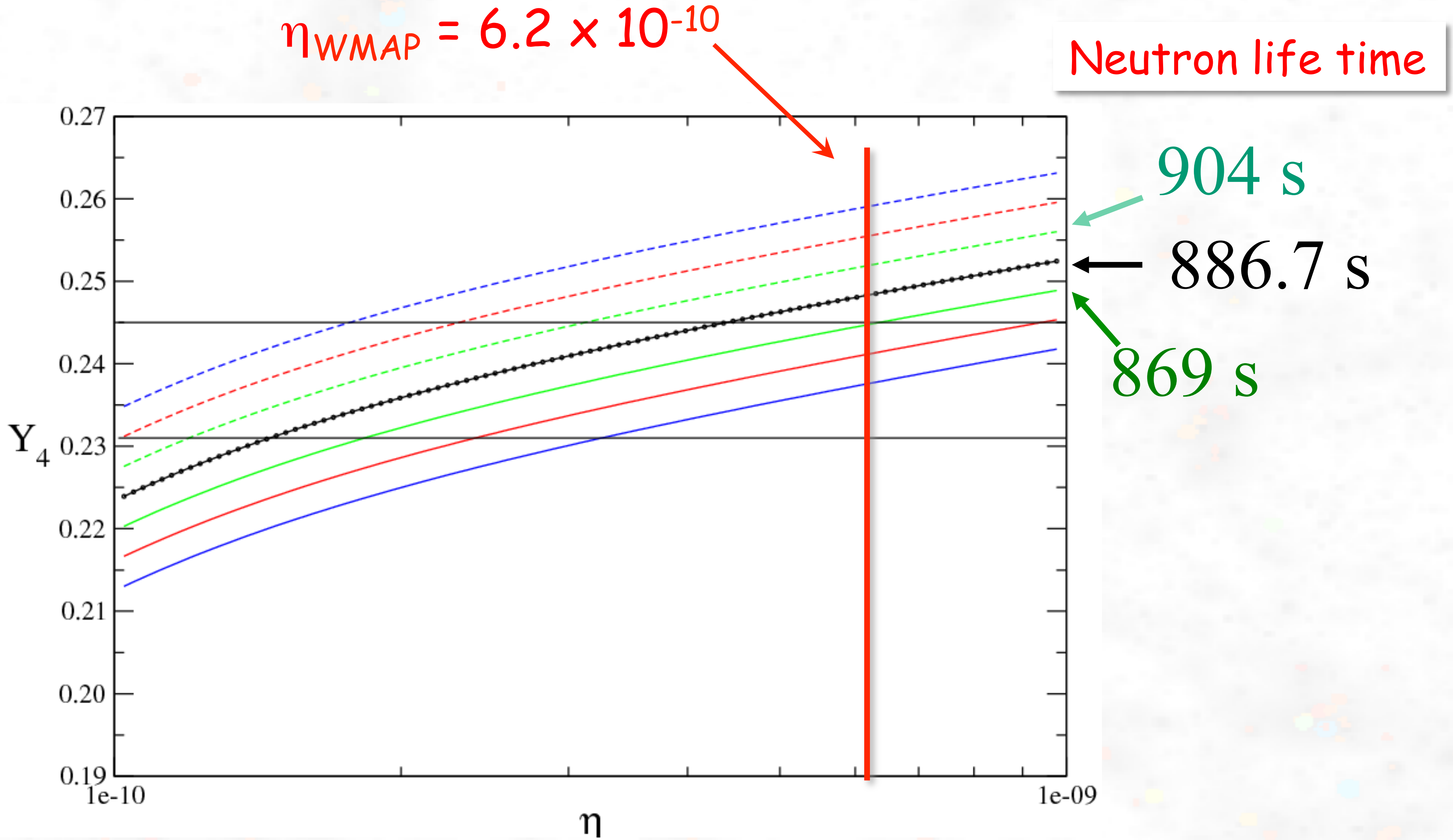
He/H ratio



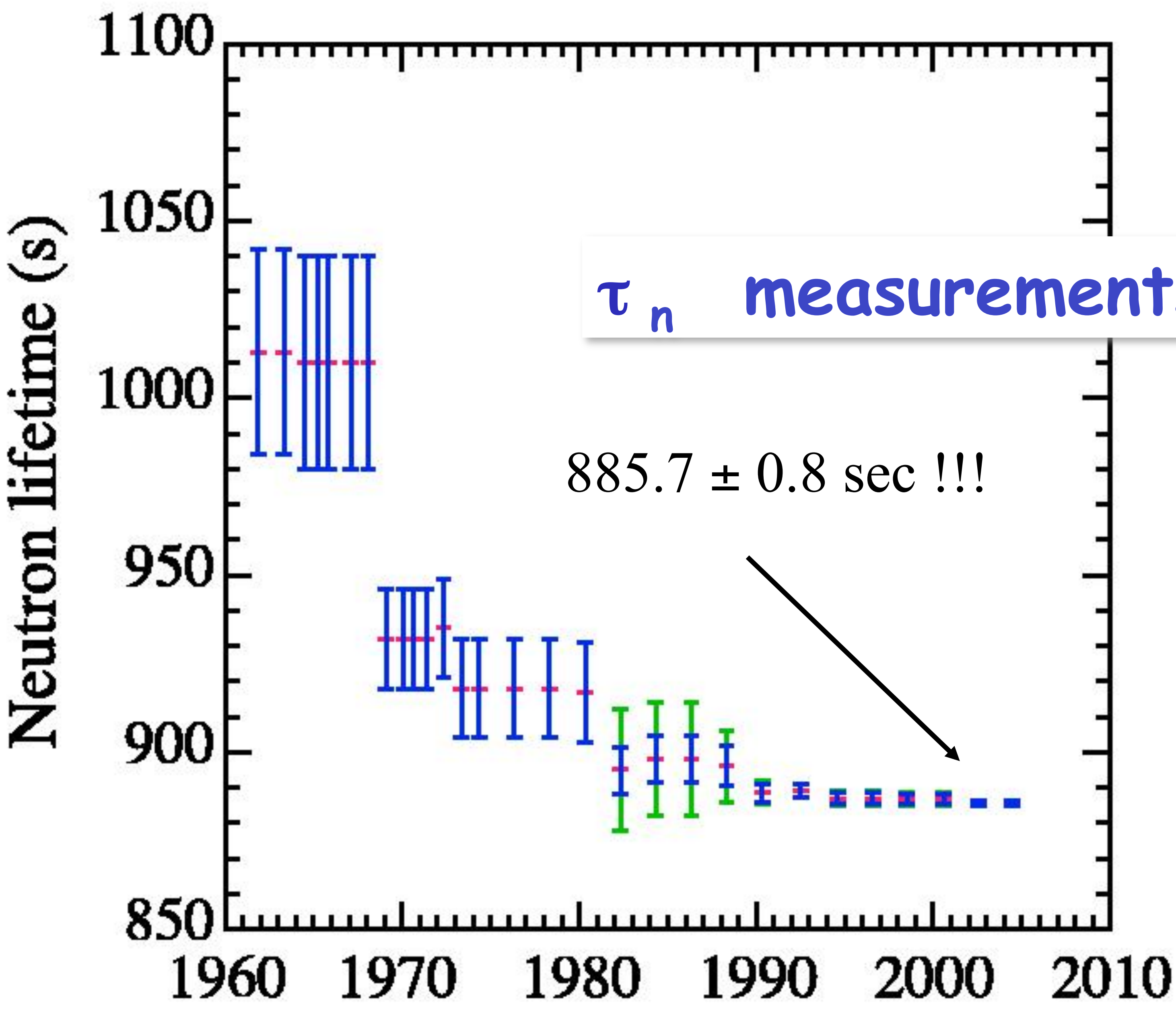
When T = 10^9 K (1 minute old), p:n = 7:1

When ^{2}H and He nuclei formed, n got within He nuclei
→ about 1 He nucleus for every 12 H nuclei
This is the abundance of He and ^{2}H we observe today

Neutron lifetime

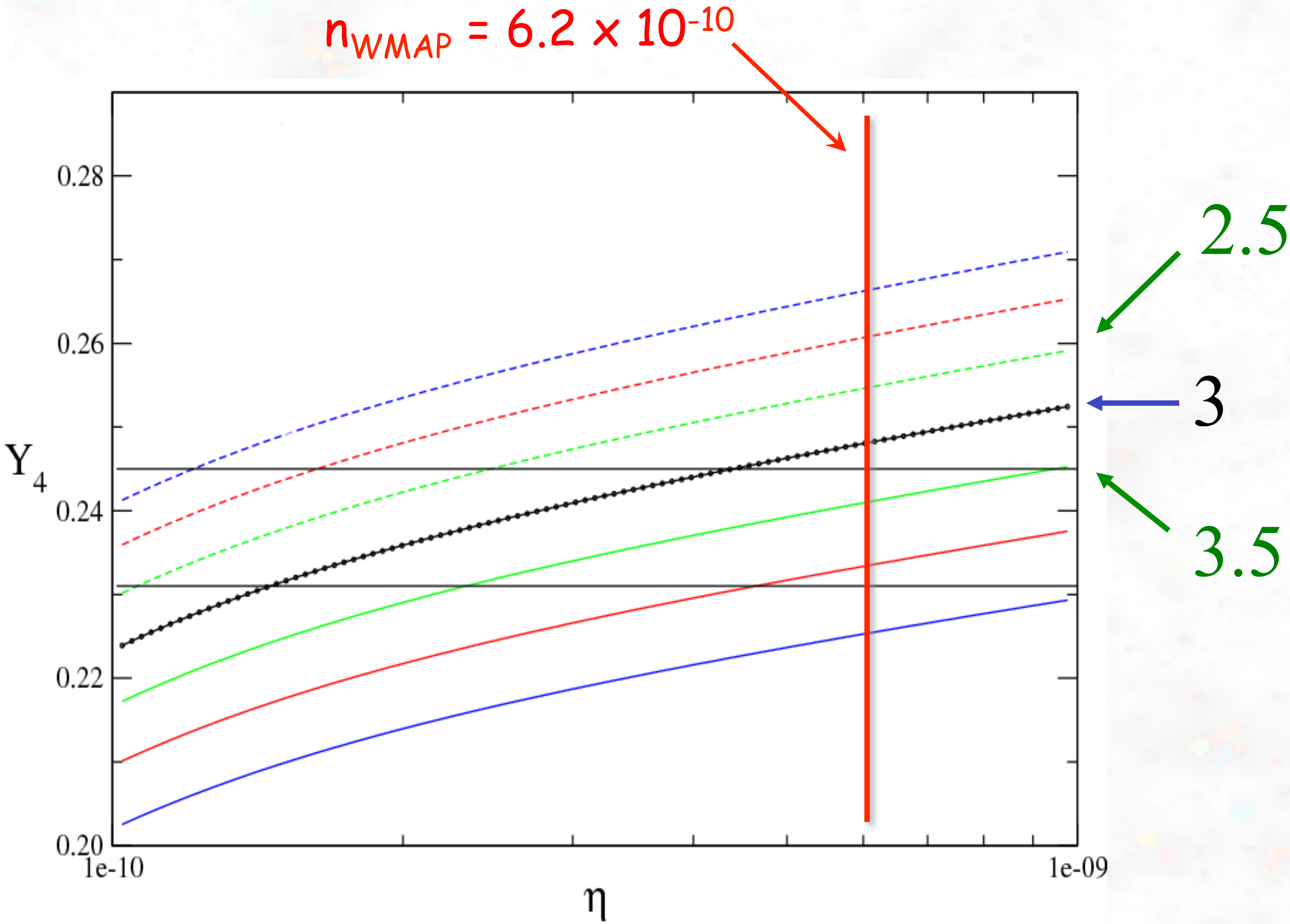


Neutron lifetime

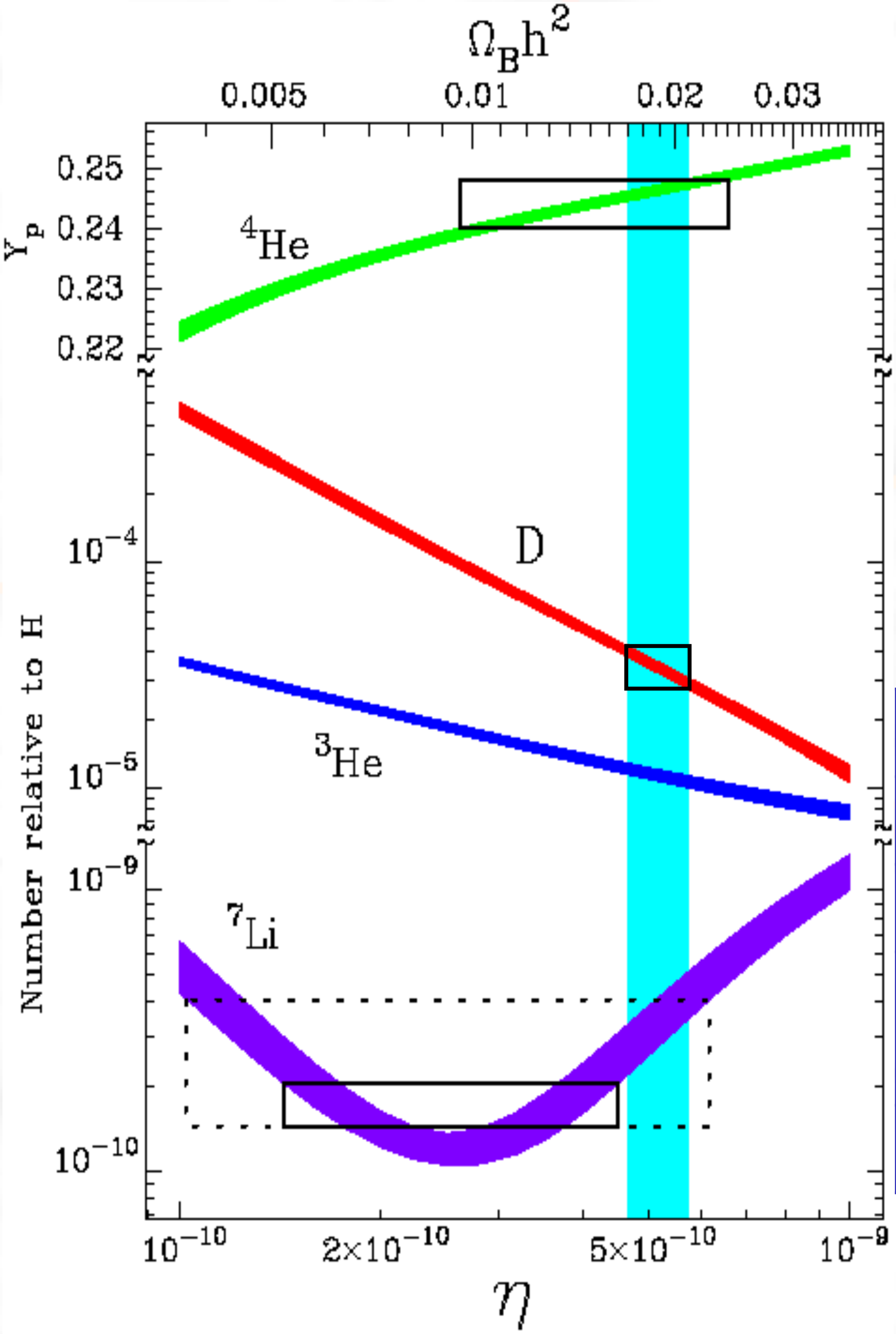


τ_n measurements vs. time

Neutrino families



BBN – incredibly successful,except for Lithium problem



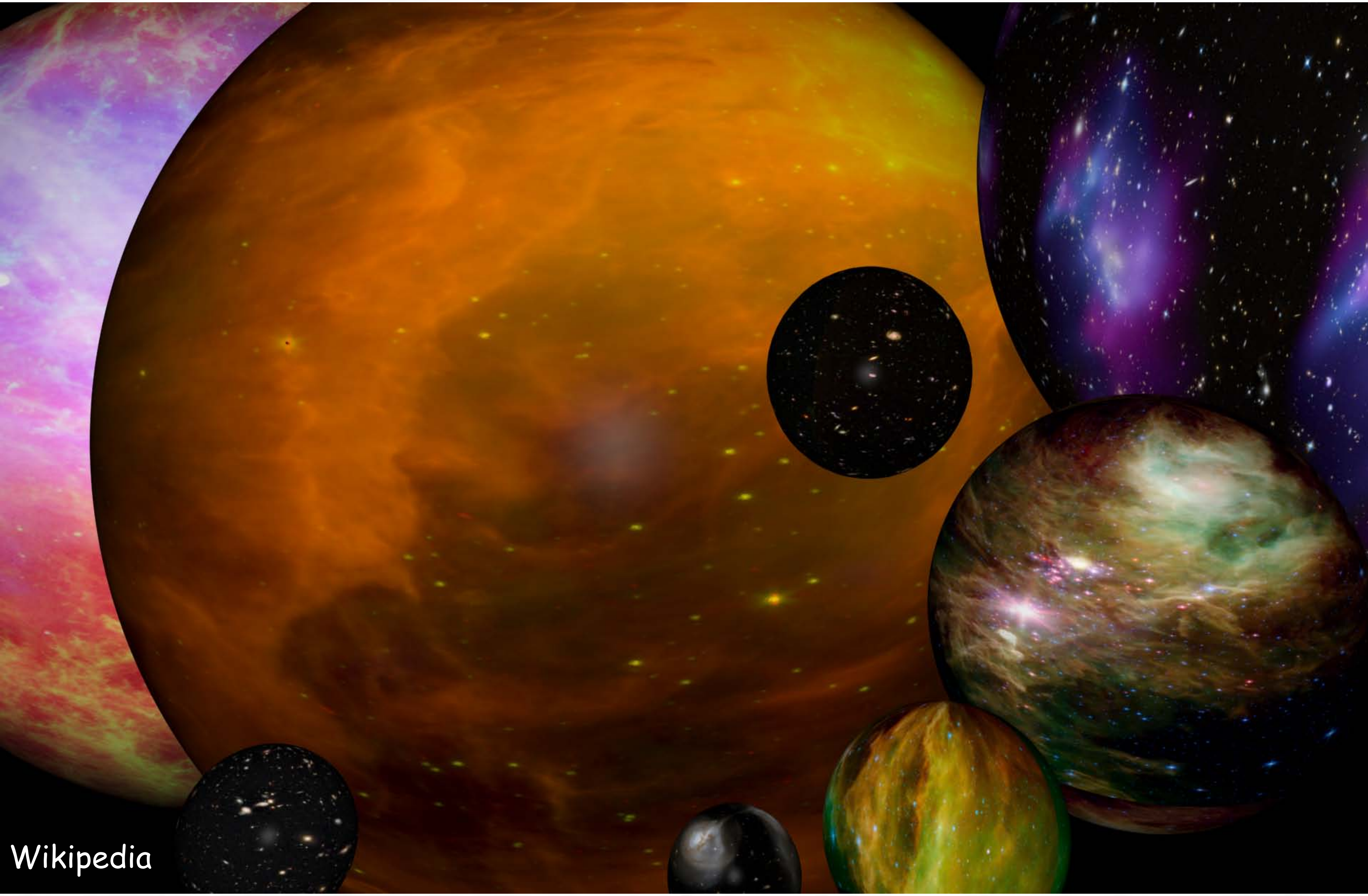
SBBN: one parameter

baryon-to-photon ratio η

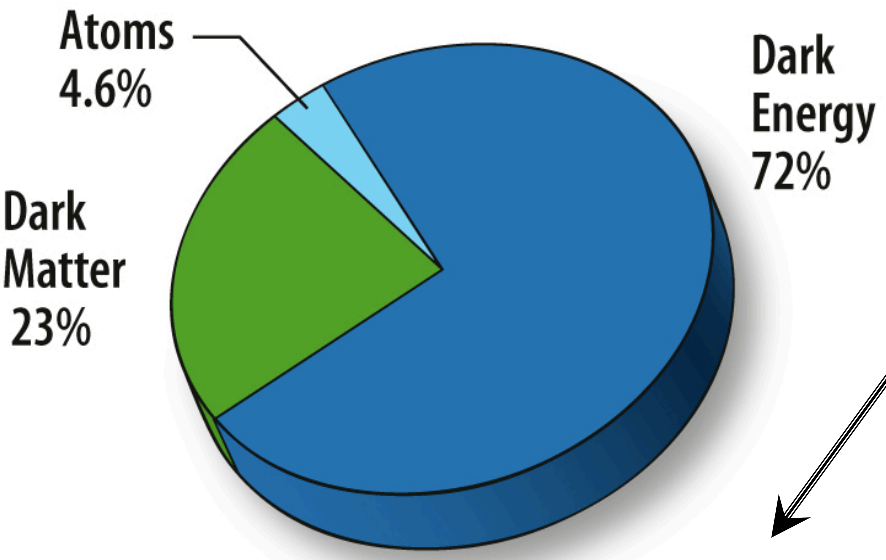
$\eta = (6.225^{+0.157}_{-0.154}) \times 10^{-10}$
(WMAP 2010)

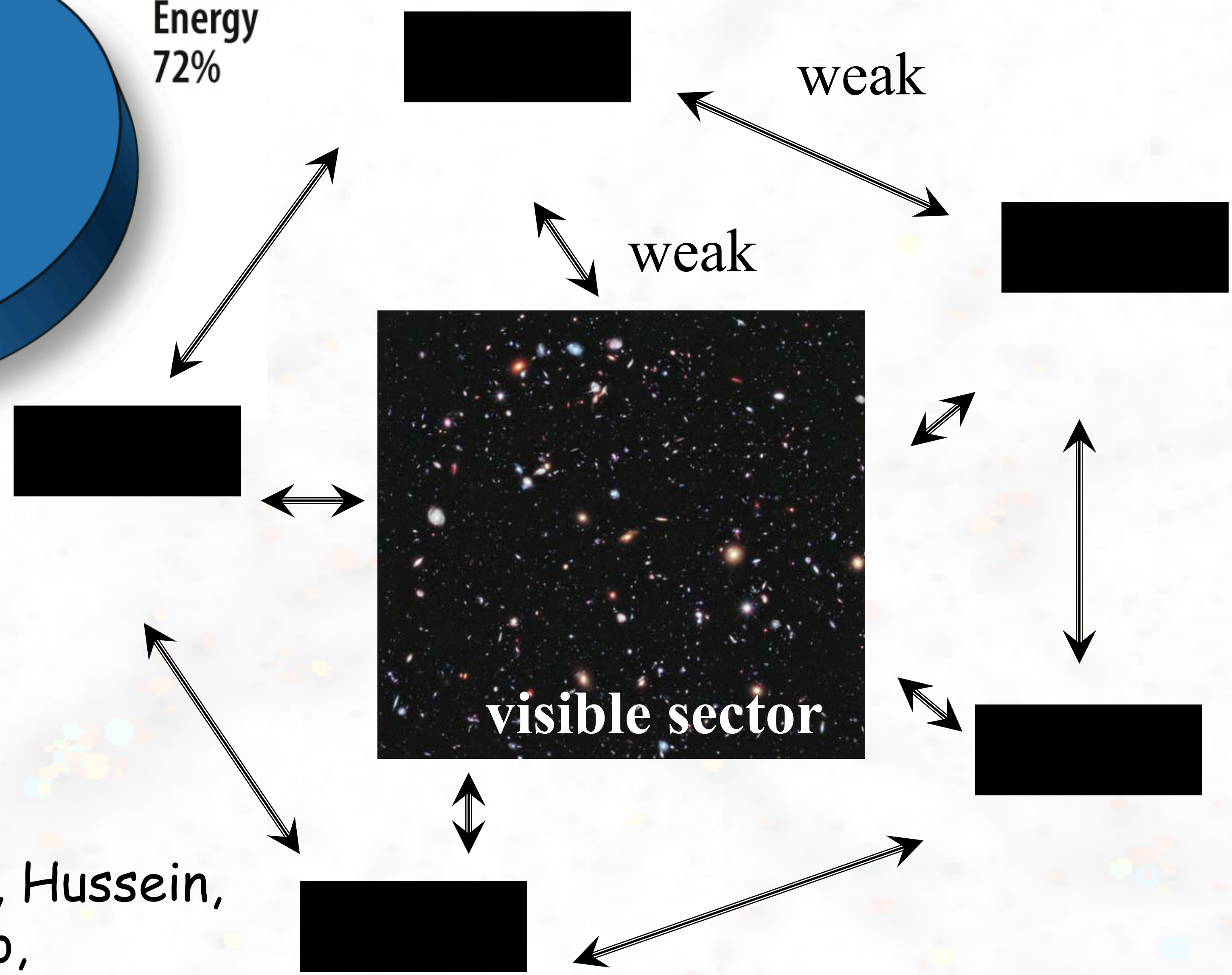
	BBN	Observation
^{4}He/H	0.249	0.249
D/H	2.62×10^{-5}	2.82×10^{-5}
^{3}He/H	0.98×10^{-5}	$(0.9\text{-}1.3) \times 10^{-5}$
^{7}LI/H	4.39×10^{-10}	1.1×10^{-10}

Parallel universes



Parallel universes of dark + visible matter

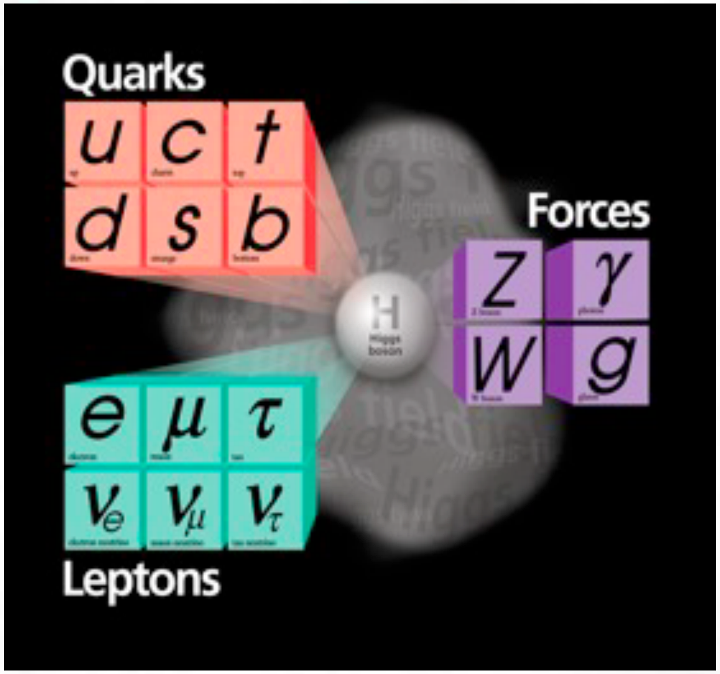




dark
sector

Oliveira, Bertulani, Hussein,
de Paula, Frederico,
arXiv:1108.2723

Parallel Universes of Dark Matter



$$Q_1 = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \qquad Q_2 = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$Q_3 = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \qquad Q_4 = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

1. \+ 5 superfamilies of particles
2. each superfamily has its own SM (dark photon, dark W, etc.)
3. new SU(3) gauge boson WIMG connecting different superfamilies

Weakly Interacting Massive Gray (**WIMG**) boson, aka **Mulato** (or Mulatto)

Mulato

Parallel Universes of Dark Matter

1. Leaves unchanged long distance properties of SM and Gravity
2. No Higgs Mechanism
3. Compatible with Cosmological constraints and BBN

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{Q}_f \left[i\gamma^\mu D_\mu - m_f \right] Q_f + \frac{1}{2} \left(D^\mu \phi^a \right)\left(D_\mu \phi^a \right) - V_{oct}\left(\phi^a \phi^a \right)$$

Mulato + **Matter Field** + **Scalar**

$$D_\mu = \partial_\mu + i g_M T^a M^a_\mu$$

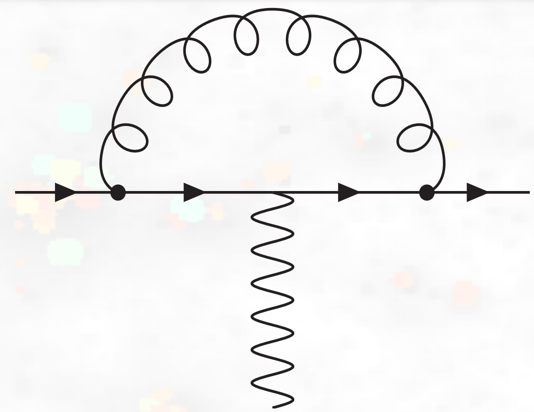
Mulato mass

ϕ^a adjoint representation of SU(3)
no coupling to U(1) groups

$$\frac{1}{2}\left(D^\mu \phi^a\right)\left(D_\mu \phi^a\right) \Rightarrow \frac{1}{2} g_M^2 \phi^c \left(T^a T^b\right)_{cd} \phi^d M_\mu^a M^{b\,\mu}$$

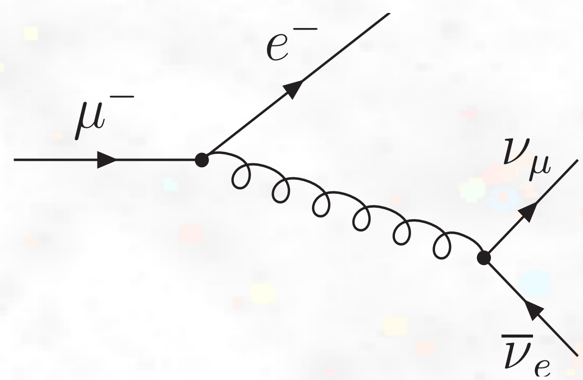
$$\left\langle \phi^a \right\rangle = 0 \qquad \left\langle \phi^a \phi^b \right\rangle = v^2 \delta^{ab}$$

$$M^2 = 3 g_M^2 v^2 \Leftarrow \text{gauge invariant}$$



Leptonic anomalous magnetic moment

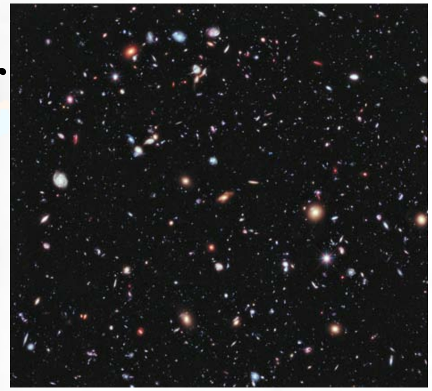
Muon beta decay



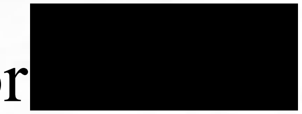
$$M \geq 9 \text{ TeV} \Leftarrow$$

Dark sectors are colder

T ordinary matter



T' dark sector



$$\rho = \frac{\pi^2}{30} g_*(T) T^4$$

$$s = \frac{2\pi^2}{45} g_s(T) T^3$$

$$g_*(T) = \sum_B g_B \left(\frac{T_B}{T}\right)^4 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T}\right)^4$$

$$g_s(T) = \sum_B g_B \left(\frac{T_B}{T}\right)^3 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T}\right)^3$$

Do it for T and T' + use Friedmann equation and evolve to BBN time

$$\bar{g}_s(T) = g_* \left[1 + N_D \left(\frac{T'}{T}\right)^4\right]$$

$$g_s(T)\big|_{T=1\,\mathrm{MeV}} = 10.75$$

BBN + ^{4}He, ^{3}He, D and ^{7}Li constraints →

$$\frac{T'}{T} < \frac{0.78}{N_D^{1/4}} = 0.52$$

Baryon asymmetry and DM halo dynamics

$$\eta = \frac{\text{density of baryons}}{\text{density of photons}}$$

$$\frac{\text{dark baryons}}{\text{ordinary baryons}} = \frac{\eta'}{\eta}\left(\frac{T'}{T}\right)^3 \sim 1 \quad \Rightarrow \quad \eta' = 7\eta$$

Acoustic oscillations → to change CMB: $\frac{T'}{T} \geq 0.6$ → $N_D \geq 0.35$

lower bound

DM-DM interactions

$$\sigma = \left(g^2 \frac{T}{\Lambda^2}\right)^2 \quad \rightarrow \quad \frac{\sigma'}{\sigma} \sim \left(\frac{T'}{T}\right)^2 \quad \xrightarrow{\text{BBN}} \quad \frac{\sigma'}{\sigma} \sim \frac{0.61}{\sqrt{N_D}}$$

Dark Sectors are essentially collisionless

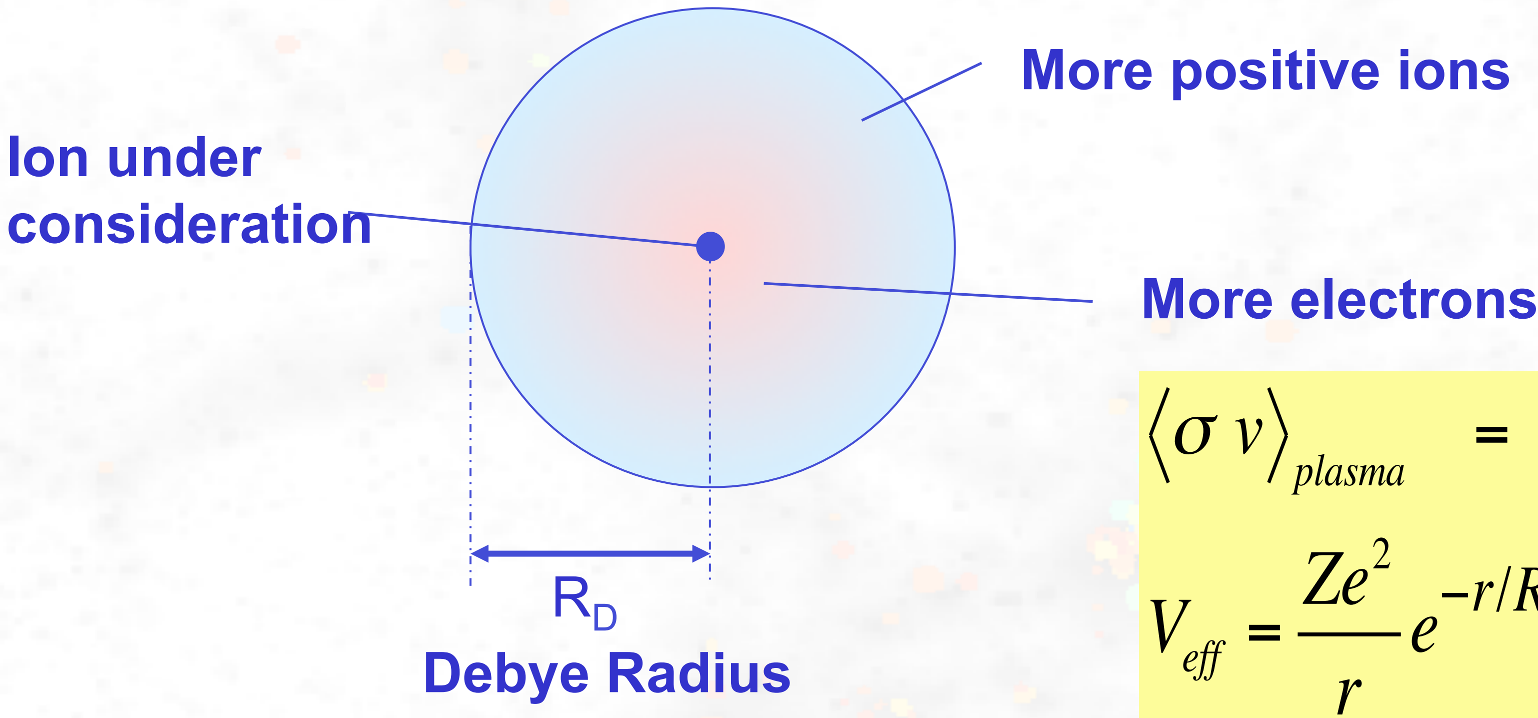
Model is compatible with cosmological, BBN and CMB constraints

Screening

Electron screening in plasmas

Debye-Hueckel screening (Salpeter 1959)

$$n\, R_D^3 >> 1$$



$$\langle \sigma\, v \rangle_{plasma} = f(E) \langle \sigma\, v \rangle_{bare}$$

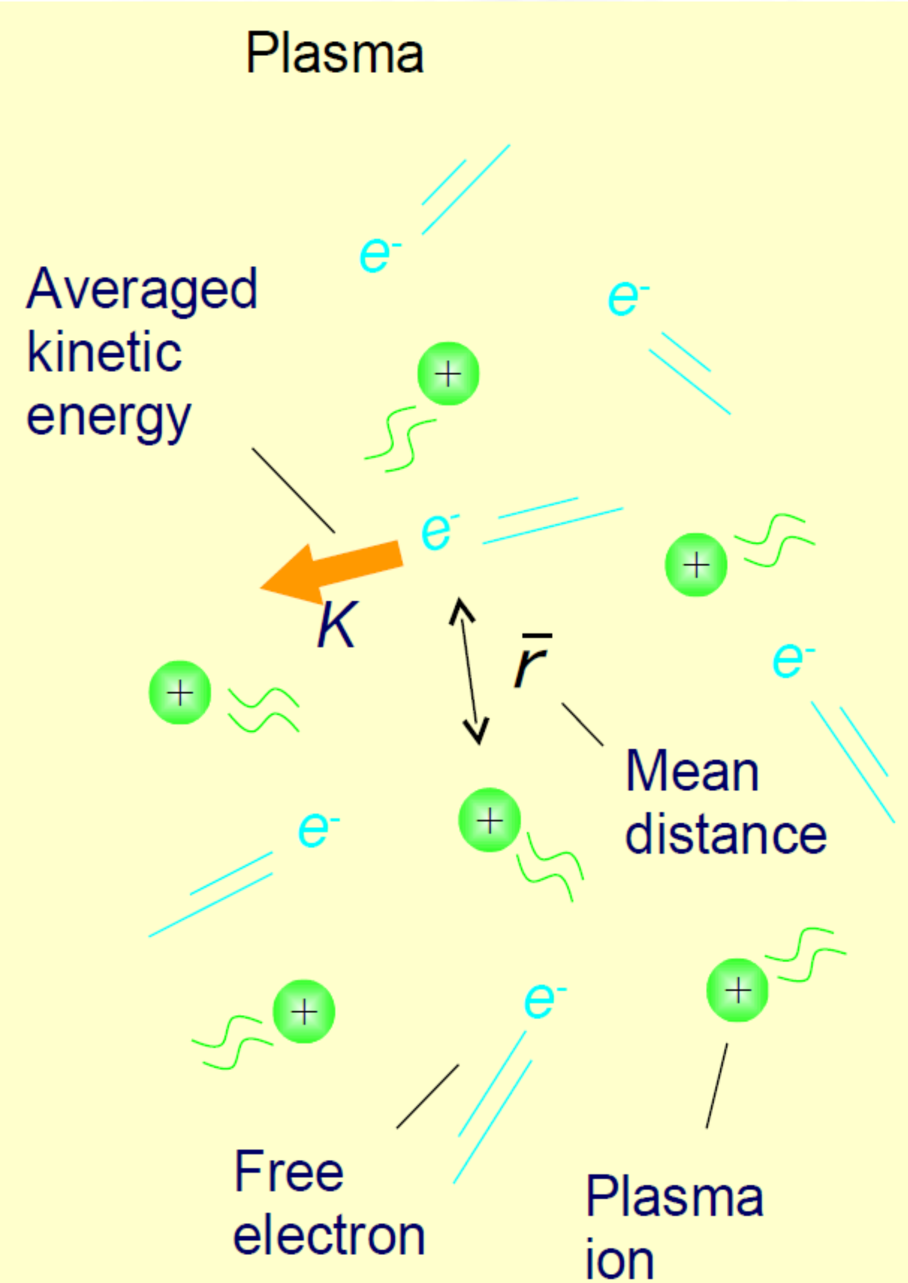
$$V_{eff} = \frac{Ze^2}{r} e^{-r/R_D}$$

$$R_D = \sqrt{\frac{kT}{4\pi e^2 \rho N_A \xi}} \sim 0.218\ \overset{0}{\mathrm{A}}\ (\mathrm{Sun})$$

$$\xi = \sum_i (Z_i^2 + Z_i)^2 Y_i$$

$$^7\mathrm{Be}(\mathrm{p},\gamma)^8\mathrm{B}\ \ (T \sim 10^7 \mathrm{K}): \qquad f(E) \sim 1.2$$ (20 % effect)

Screened big bang



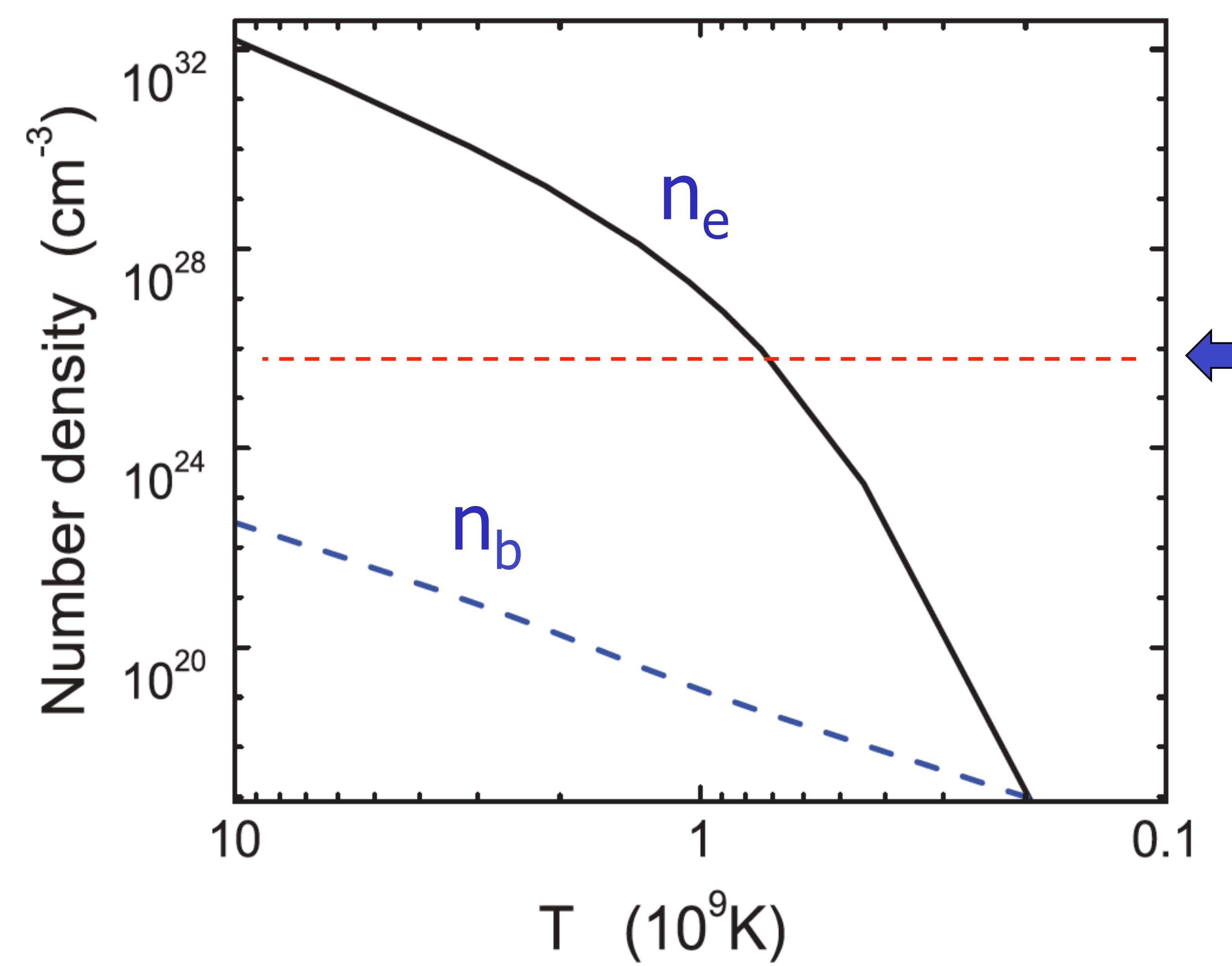
- Interactions with photons/ electrons of the plasma
- Change in the e.m. equation of state due to photon/ electron thermal masses

P=P(ρ)

Wang, Bertulani, Balantekin, Phys. Rev. C 83, 018801 (2011)

Electron density in the big bang

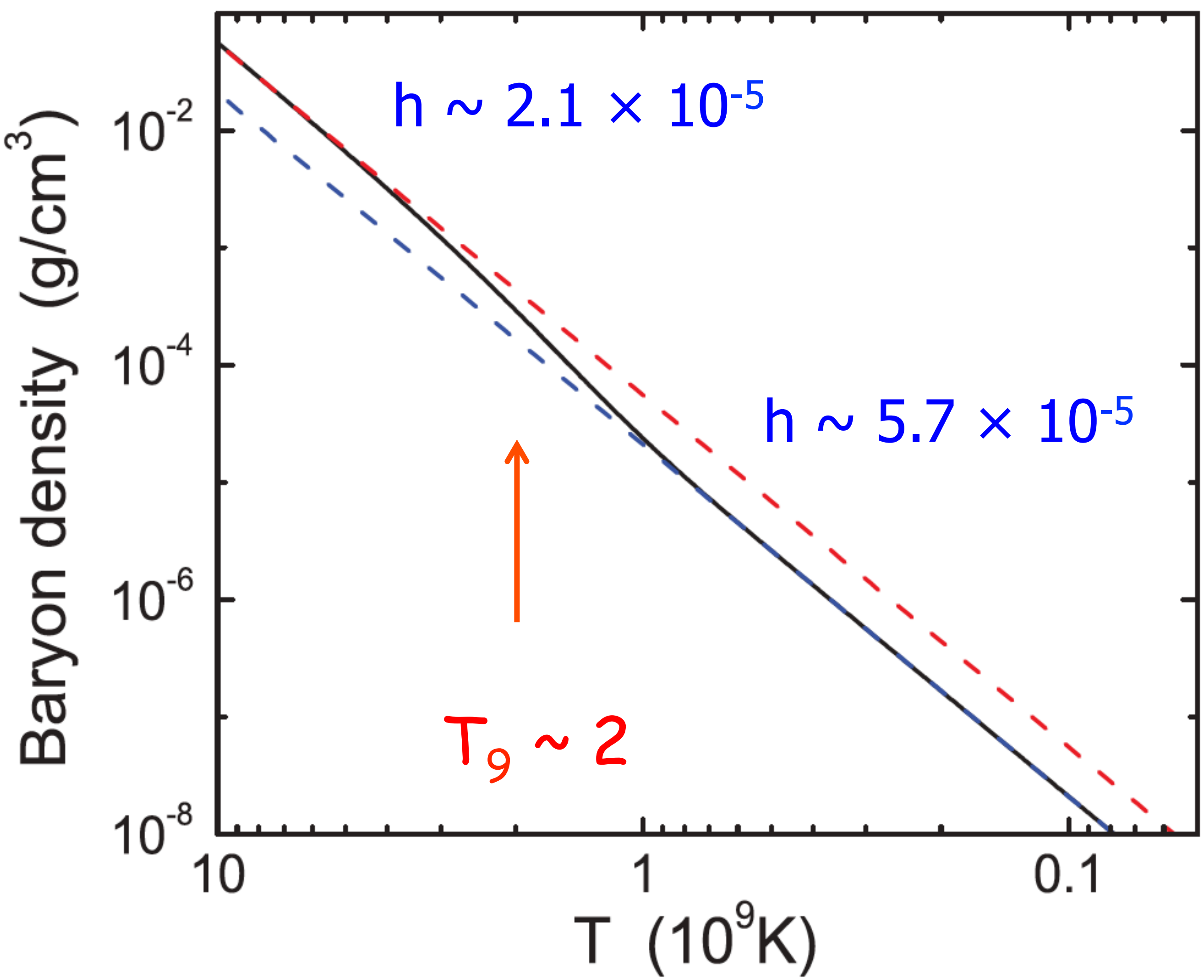
mostly due to $\gamma \rightarrow e^+e^-$



n_e center of sun

n_e(BB) ~ n_e (sun) **but** n_b(sun) > n_b (BB)

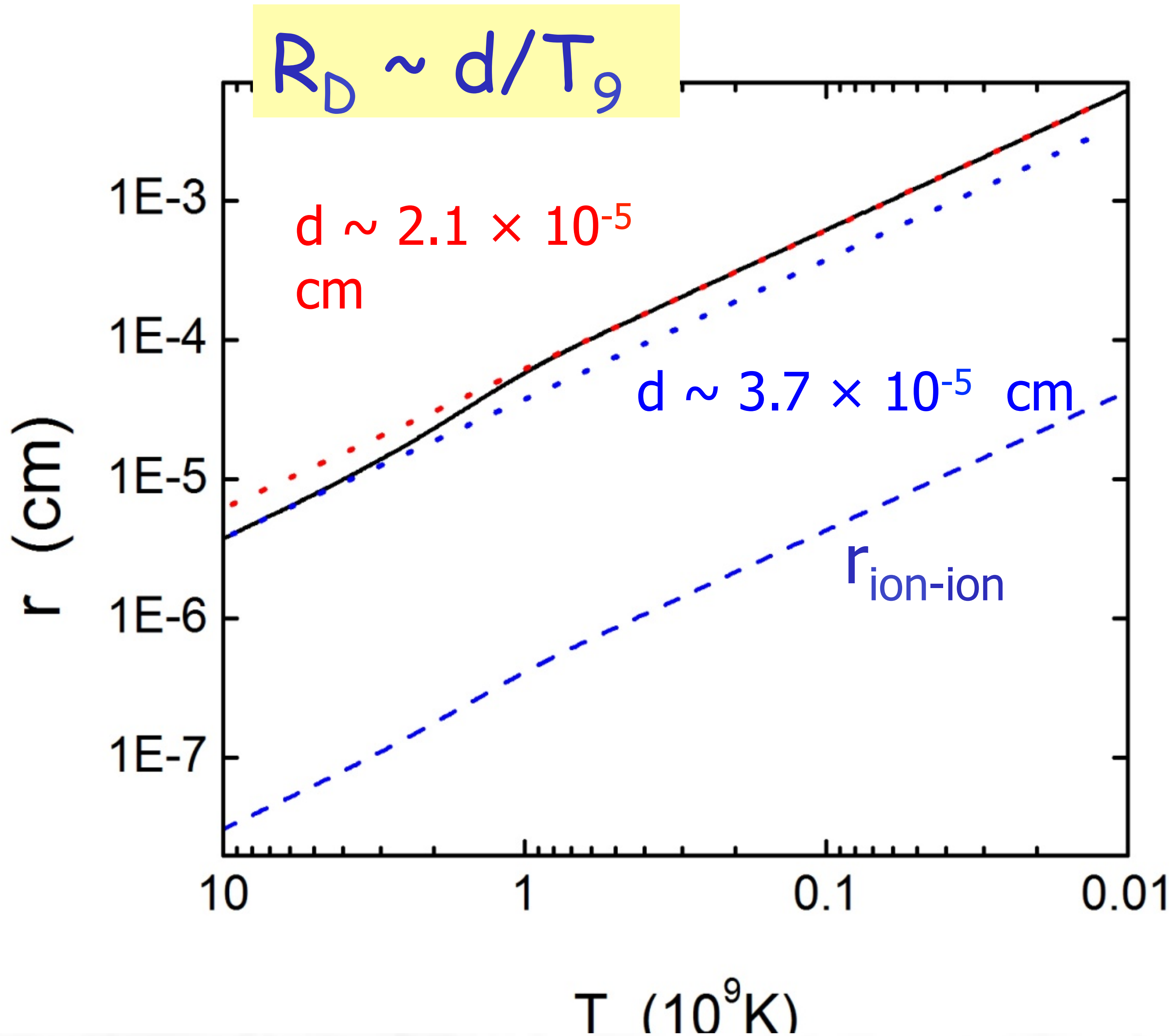
Baryon density



$\rho_b \sim h T_9^3$

degrees of freedom changes around $T_9 \sim 1$

Debye radius x inter-ion distance



≥ 1000 particles in Debye sphere

mean-field model OK

Debye model + dynamical corrections

Debye-Hueckel in the big bang

$$f_{BBN} = \exp(4.49\times10^{-8}\xi Z_1 Z_2) \quad \text{for } T_9 < 1$$
$$= \exp(2.71\times10^{-8}\xi Z_1 Z_2) \quad \text{for } T_9 \geq 1$$

$$\xi = \sum_i (Z_i^2 + Z_i)^2 Y_i$$

Extended Debye-Hueckel: dynamical correlations

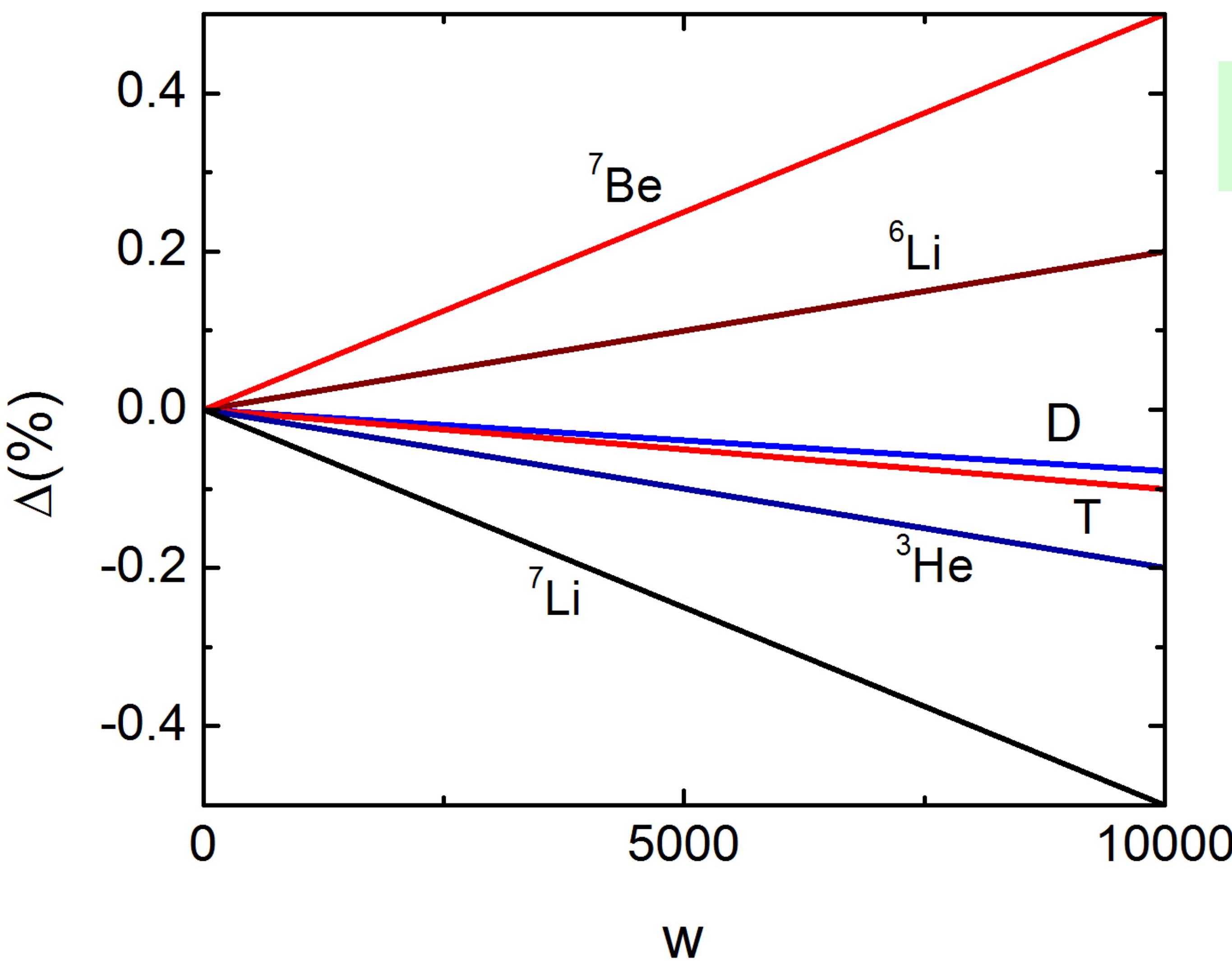
N. Itoh et al, Ap. J., 488, 507 (1997)

< 1% effect on BBN

Simplified account of dynamical corrections

$$R_D = \sqrt{R_D^{(0)}\left(1+\frac{E}{2kT}\right)}$$

If screening effect was much larger

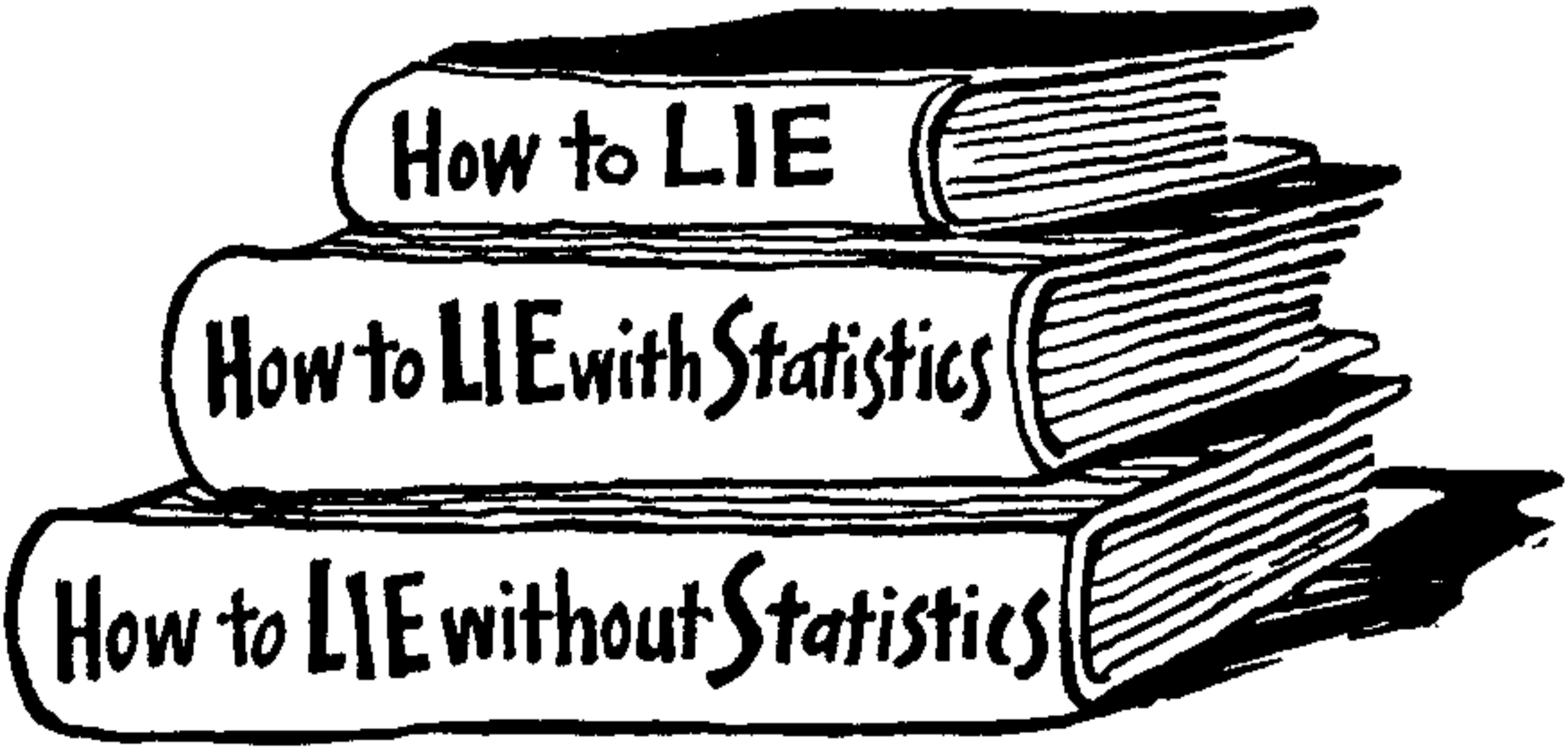


$$\ln f_{BBN}^{'} = w \ln f_{BBN}$$

$$\Delta = \frac{Y' - Y}{Y} \times 100$$

BBN screening of reaction rates by electrons negligible

Statistics

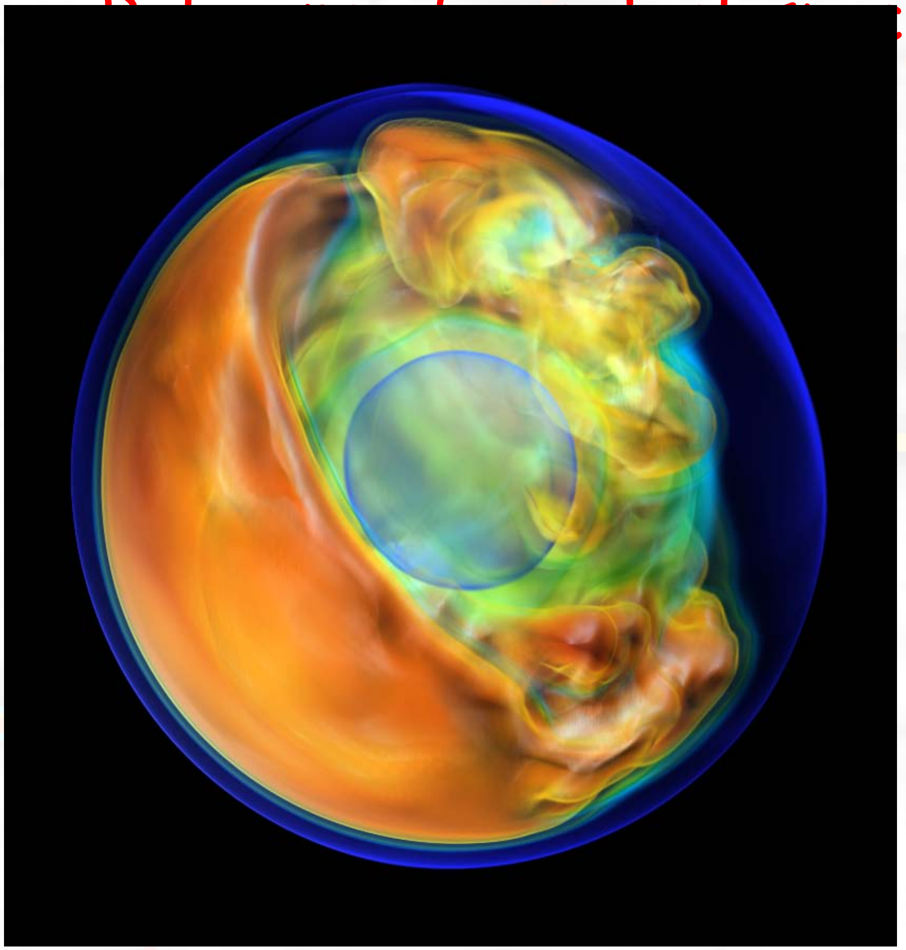


How to LIE
How to LIE with Statistics
How to LIE without Statistics

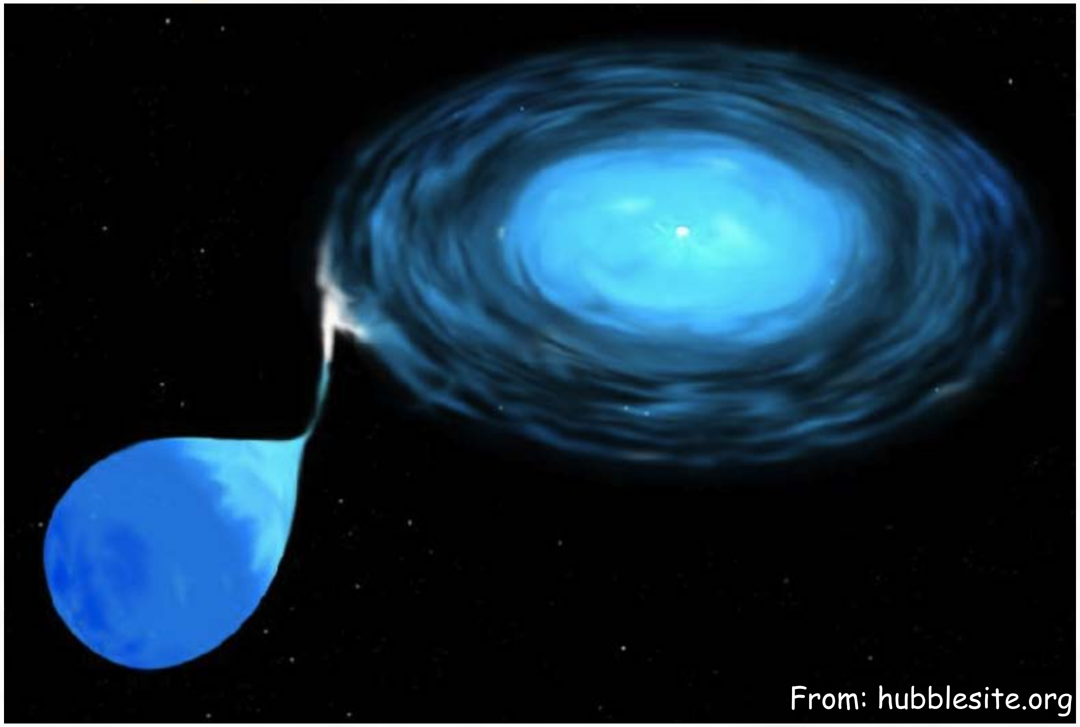
Non-Maxwellian thermal distribution

- Deviation from Boltzmann-Gibbs statistics now very popular in plasma physics → turbulence phenomena, systems having memory effects, systems with long range interactions, etc.

- … stellar systems, e.g. supernovae?



Courtesy: (ORNL)



Extensive statistics

Standard Boltzmann-Gibbs statistics $$S = -k_B \sum_{i=1}^{n} p_i \ln p_i$$

Has a maximum when all states have equal probability p_i

Two simple constraints (normalization and mean value of the energy):

$$\int_0^\infty p(\varepsilon)\, d\varepsilon = 1$$

$$\int_0^\infty \varepsilon\, p(\varepsilon) d\varepsilon = const$$

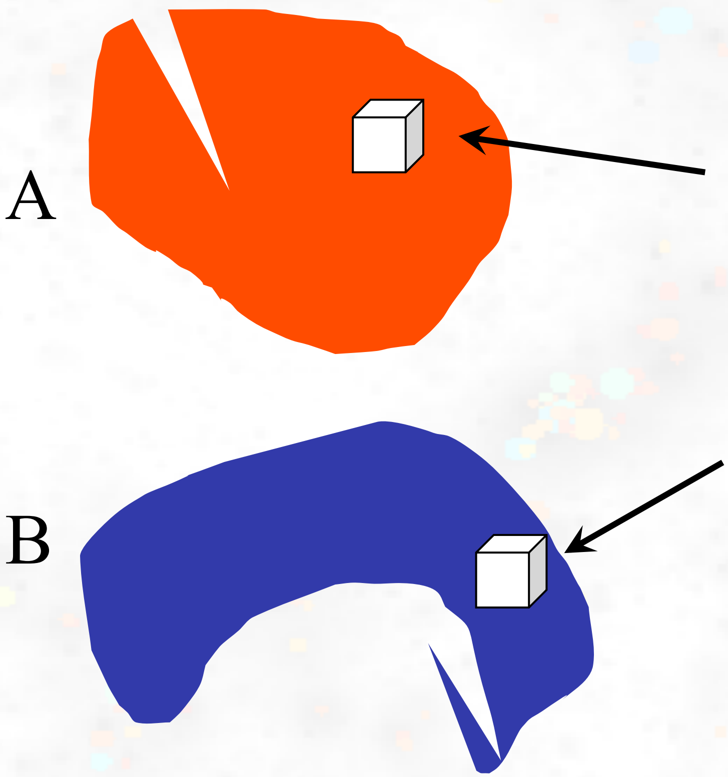
distribution function: $$p_i = \frac{e^{-\beta \varepsilon_i}}{\sum_{i=1}^{n} e^{-\beta \varepsilon_i}}$$ $\beta = 1 / k_B T$ thermodynamics

Extensive statistics

$$S = -k_B \sum_{i=1}^{n} p_i \ln p_i$$

Assumption:	particles independent	→	no correlation
Hypothesis:	**isotropy of velocity directions**	**→**	**extensivity**

+ microscopic interactions short ranged, Euclidean space time, etc.



$$p_i^A$$

$$p_j^B$$

$$p_{i+j}^{A+B} = p_i^A \cdot p_j^B$$

$$S_{A+B} = S_A + S_B$$

extensive

Non-extensive statistics

Extensive statistics not applicable for long-range interactions

THUS

→ introduce correlations via non-extensive statistics

→ derive corresponding power-law distribution

Renyi, 1955 - Tsallis, 1985

$$S_q = k_B \frac{1 - \sum_{i=1}^{n} p_i^q}{1-q}$$

$$S_q(A+B) = S_q(A) + S_q(B) + \frac{(1-q)}{k_B} S_q(A) S_q(B)$$

departure from extensivity

Non-extensive statistics

$$S_q = k_B\left(1-\sum_{i=1}^{n} p_i^q\right)/(q-1)$$

- q is *Tsallis parameter:* in general labels an infinite family of entropies

- S_q is a natural generalization of Boltzmann-Gibbs entropy which is restored for

$$\lim_{q \to 1} S_q = -k_B \sum_{i=1}^{n} p_i \ln p_i$$

- BG formalism yields exponential equilibrium distributions, whereas non-extensive statistics yields (asymptotic) power-law distributions

- Renyi entropy is related through a monotonic function to the Tsallis entropy (with k_B = 1)

$$S_q^R = \ln\left(\sum_{i=1}^{n} p_i^q\right)/(1-q) = \ln\left[1-(1-q)S_q\right]/(1-q)$$

Non-extensive statistics

$$S_q = k_B \left(1 - \sum_{i=1}^{n} p_i^q\right) / (q-1)$$

Some applications of the generalized statistics

General	M. Gell-Mann, C. Tsallis, Oxford Press, 2004
Turbulence	M. Mehrafarin, PRE 84, 022102 (2011)
High energy collisions	Biro, Purcsel, Urmossy, EPJ A40, 325 (2009)
Cosmic rays	C. Beck, EPJ. A40, 267 (2009)
Econophysics	L. Borland, PRL 89, 098701 (2002)
Biology	M.A. Moret, Physica A 390, 3055 (2011)
Networks	X. Geng and Q. Li, Physica A 356, 554 (2005)

Tsallis Statistics – Why does it matter?

* from Google Scholar March/2013

Citation indices

	All	Since 2008
Citations	18322	6918
h-index	57	35
i10-index	199	113

Citations to my articles

Select: All, None Export Show: 20 1-20 Next >

Title / Author	Cited by	Year
Possible generalization of Boltzmann-Gibbs statistics C Tsallis Journal of statistical physics 52 (1), 479-487	4150	1988
The role of constraints within generalized nonextensive statistics C Tsallis, RS Mendes, AR Plastino Physica A: Statistical Mechanics and its Applications 261 (3), 534-554	935	1998
Generalized statistical mechanics: connection with thermodynamics EMF Curado, C Tsallis Journal of Physics A: Mathematical and General 24 (2), L69	778 *	1999
Nonextensive statistics: theoretical, experimental and computational evidences and connections C Tsallis	555	1999

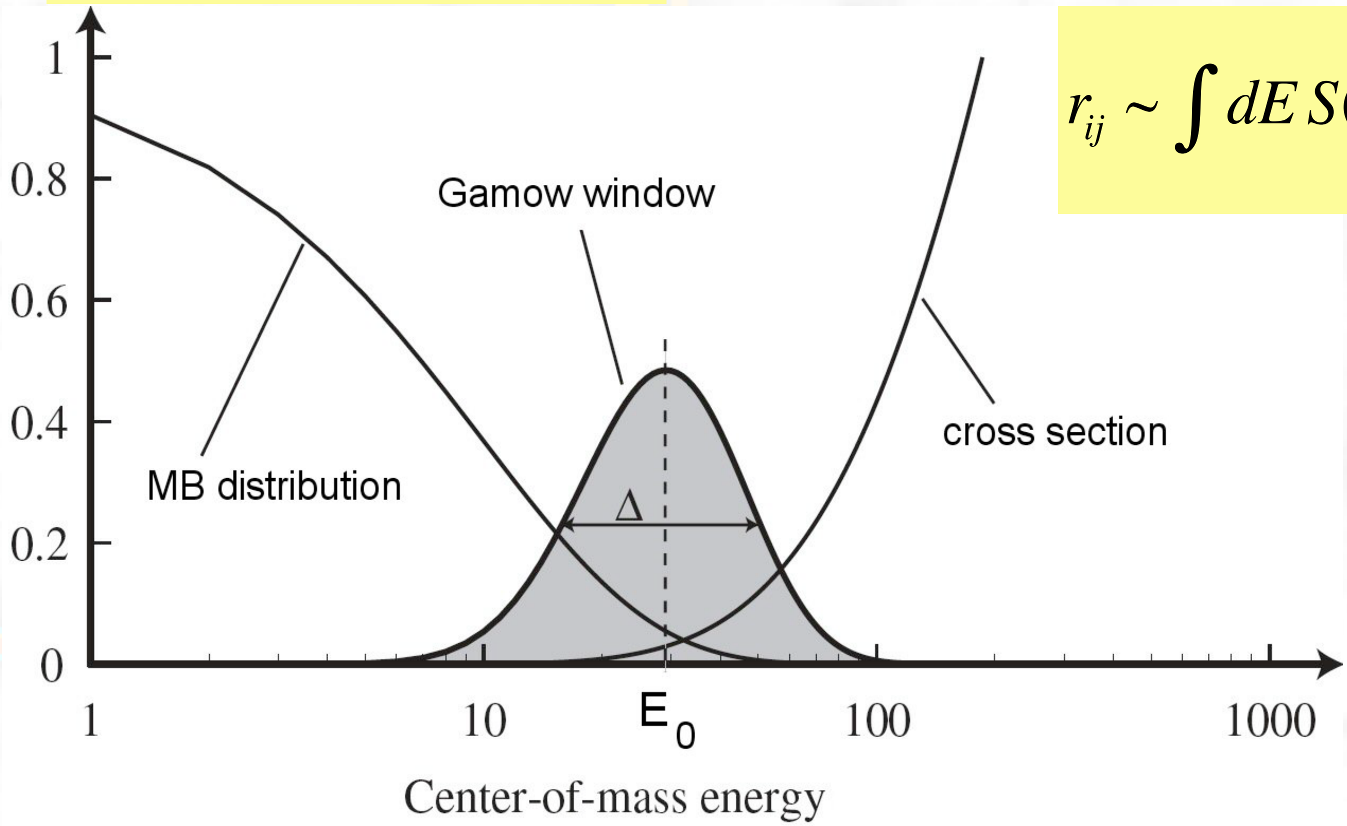
Maxwell distribution & reaction rates

$$r_{ij} = \frac{n_i n_j}{1+\delta_{ij}} \langle \sigma v \rangle$$

$$\sigma = \frac{S(E)}{E} \exp\left[-\frac{Z_i Z_j}{\hbar v} \right]$$

Bertulani, Fuqua, Hussein, Astrophys. J., (2013)

$$r_{ij} \sim \int dE\, S(E) \exp\left[-\left(\frac{E}{k_B T} \right) + 2\pi\eta(E) \right]$$



Non-Maxwellian distribution

$$f(E) = \exp\left[-\frac{E}{k_B T}\right] \quad \rightarrow \quad f_q(E) = \left(1 - \frac{q-1}{k_B T} E\right)^{\frac{1}{q-1}}$$

$$\lim_{q \rightarrow 1} f_q \quad \rightarrow \quad \exp\left[-\frac{E}{k_B T}\right]$$

$$0 \le E \le \frac{k_B T}{q-1}, \qquad if\ q \ge 1$$

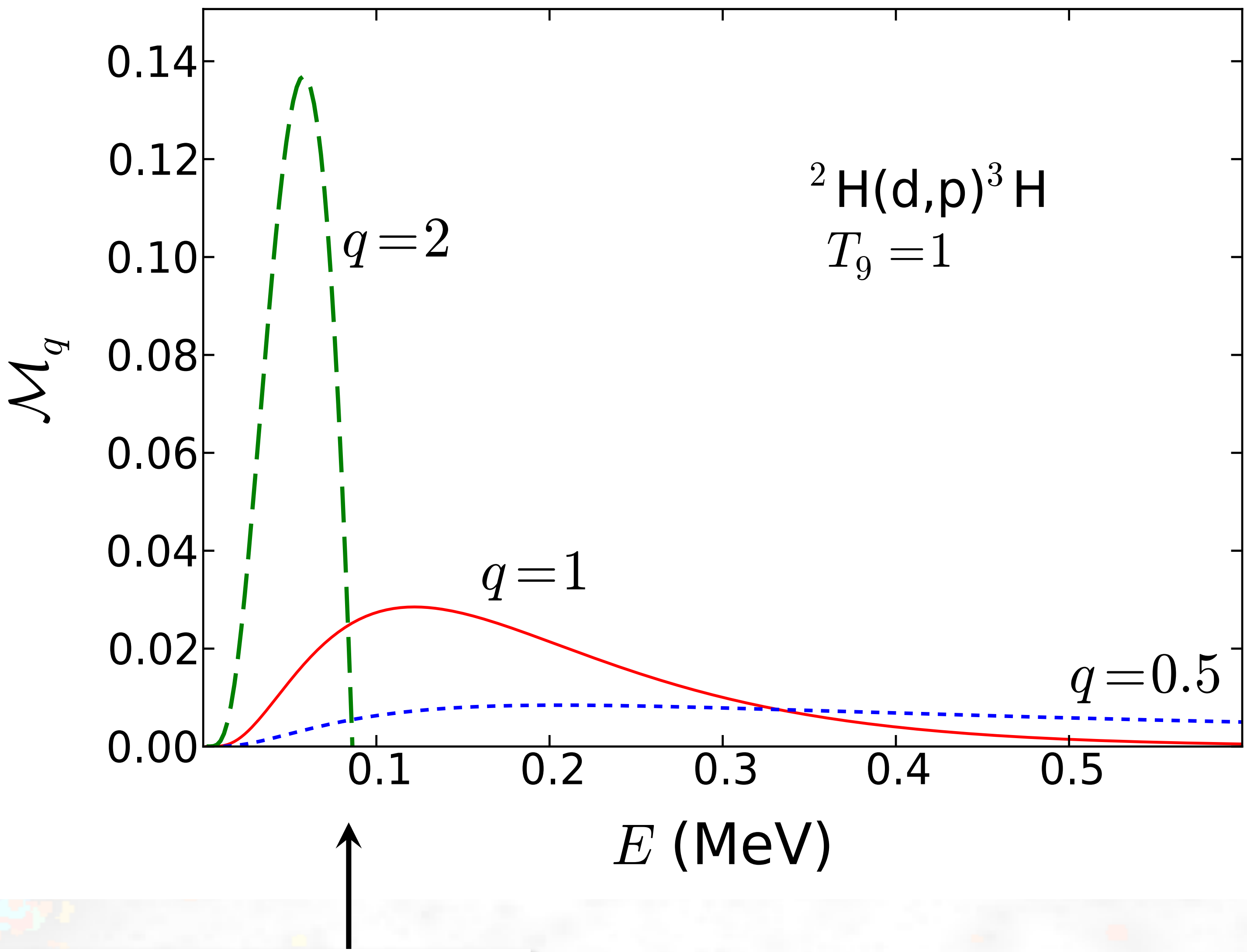
$$0 \le E \le \infty, \qquad if\ q \le 1$$

$$r_{ij} \sim \int dE\, S(E) M_q(E,T)$$

Non-Maxwellian rate

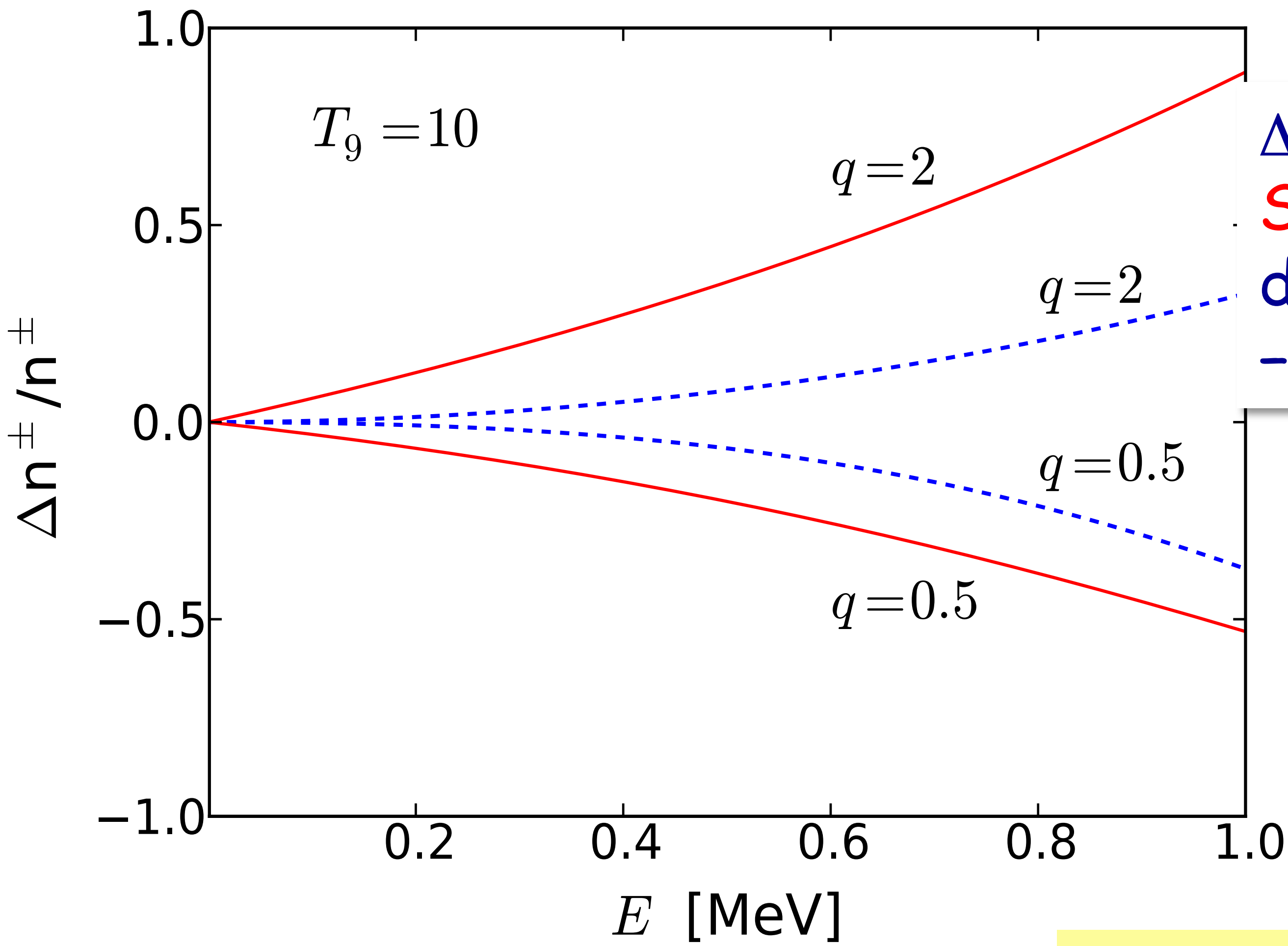
$$M_q(E,T) = \mathrm{N}(q,T)\left[1 - \frac{q-1}{k_B T} E\right]^{\frac{1}{q-1}} \exp\left[-\eta(E)\right]$$

Non-Maxwellian rates



0.086 × T_9 (MeV)
cutoff for q = 2

Non-Maxwellian rates equilibrium with e, γ, and ν



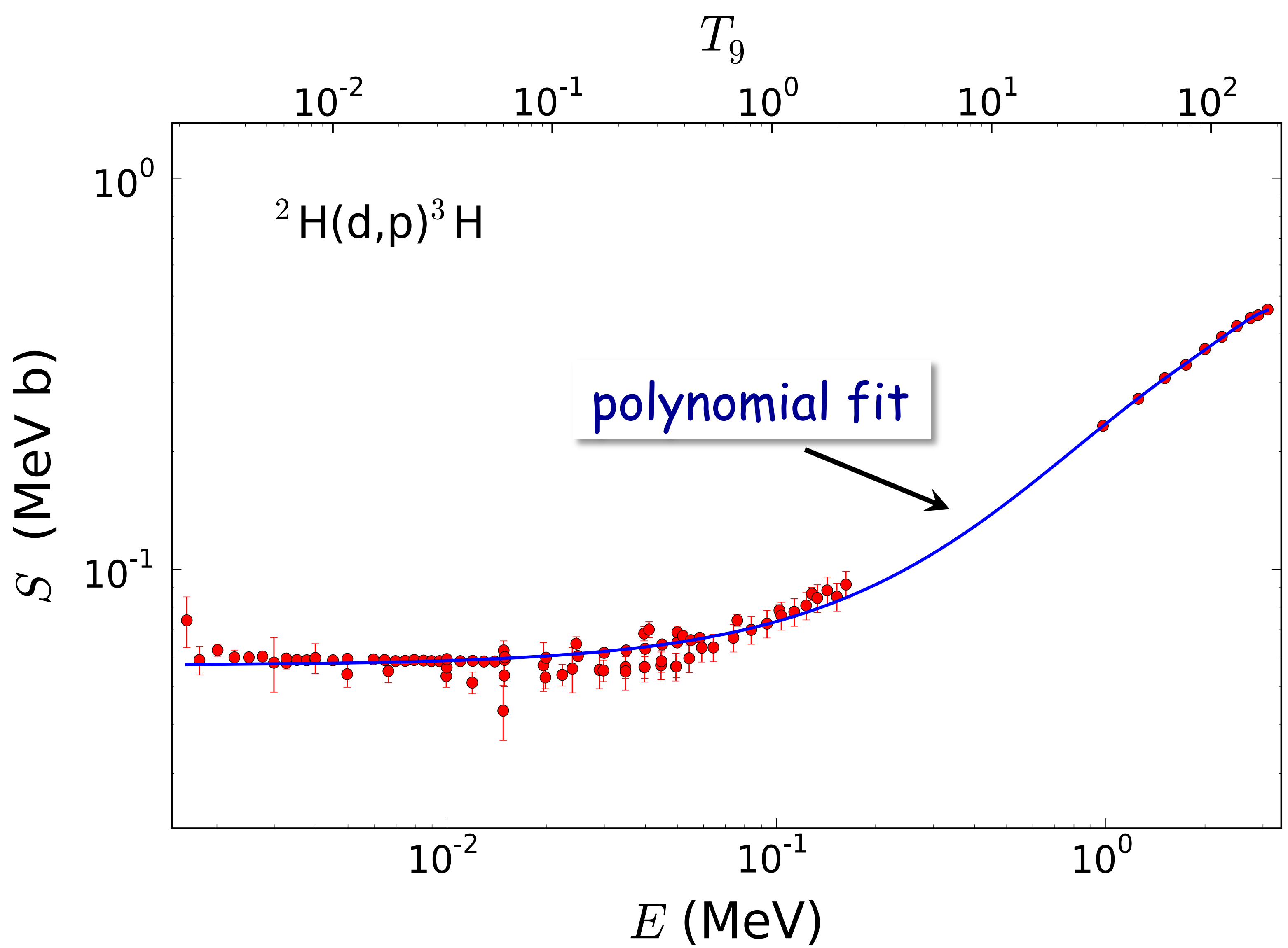
Δn/n = (n_q −n)/n.

Solid = Fermi-Dirac, n+

dashed = Bose-Einstein, n −

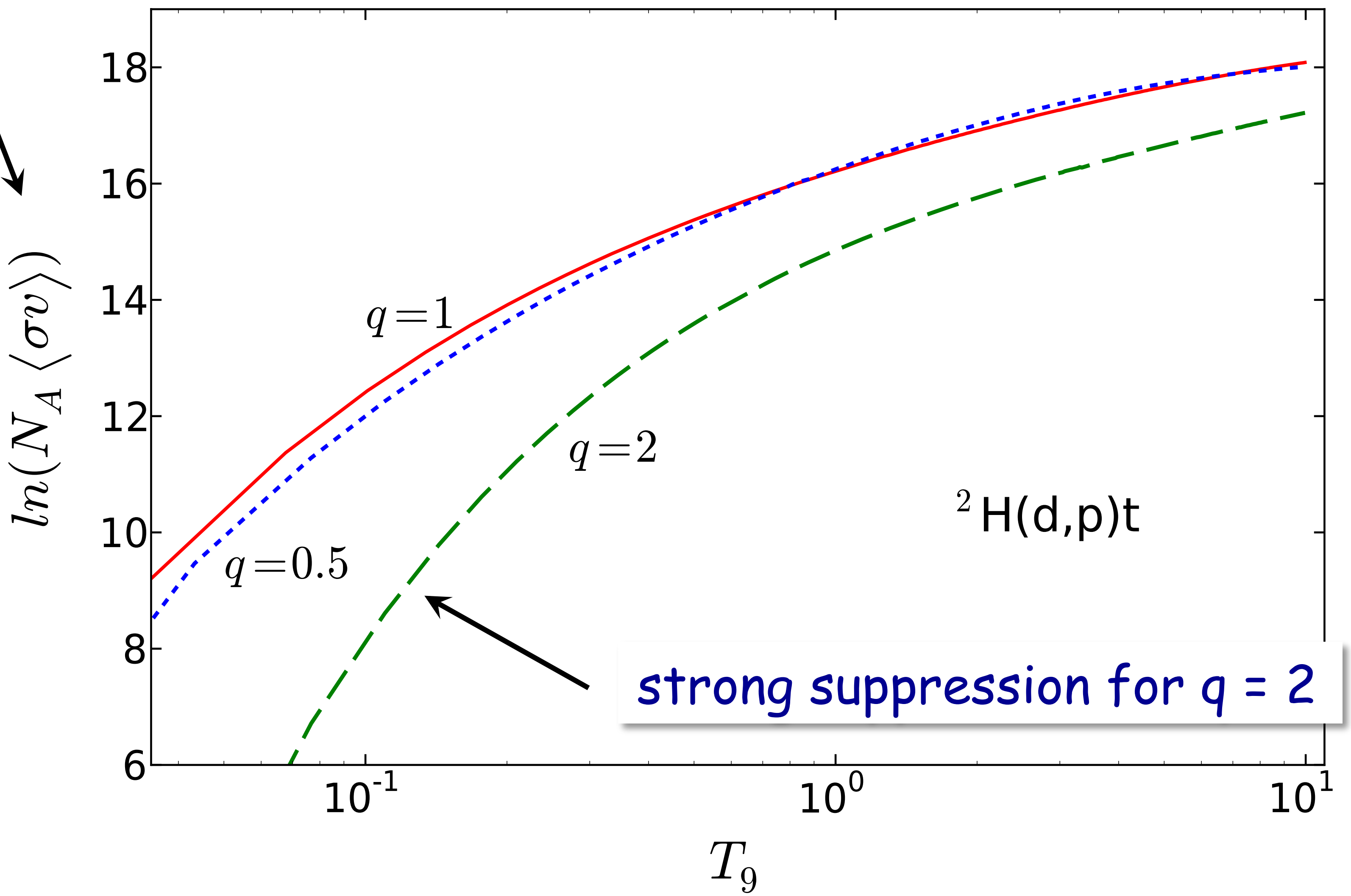
$$n_q^{\pm}(E) = \frac{1}{\left[1 - \frac{q-1}{k_B T}(E-\mu)\right]^{\frac{1}{q-1}} \pm 1}$$

Charged particles: S-factor



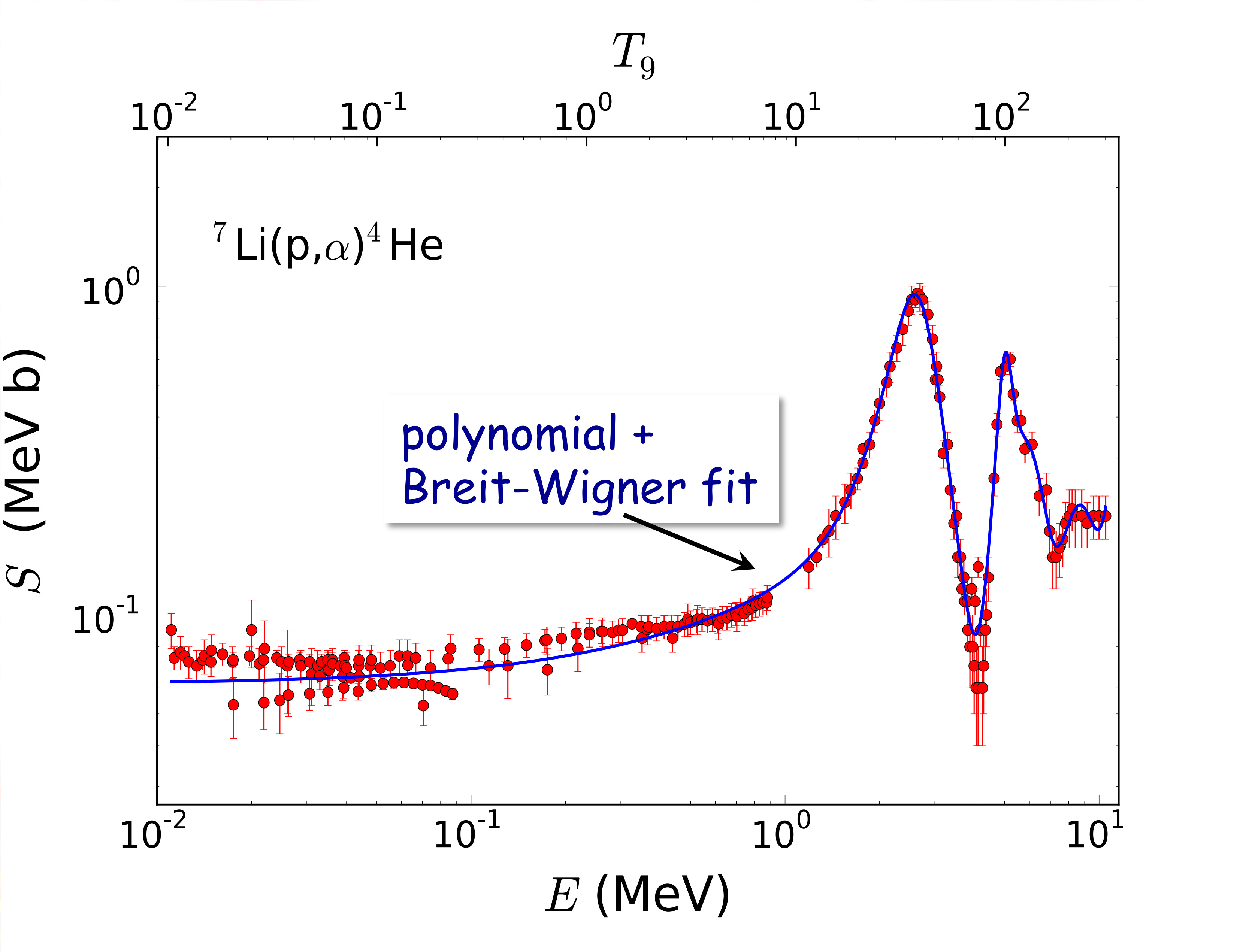
Charged particles: reaction rate

$r_{ij}\ (cm^3/mol/s)$



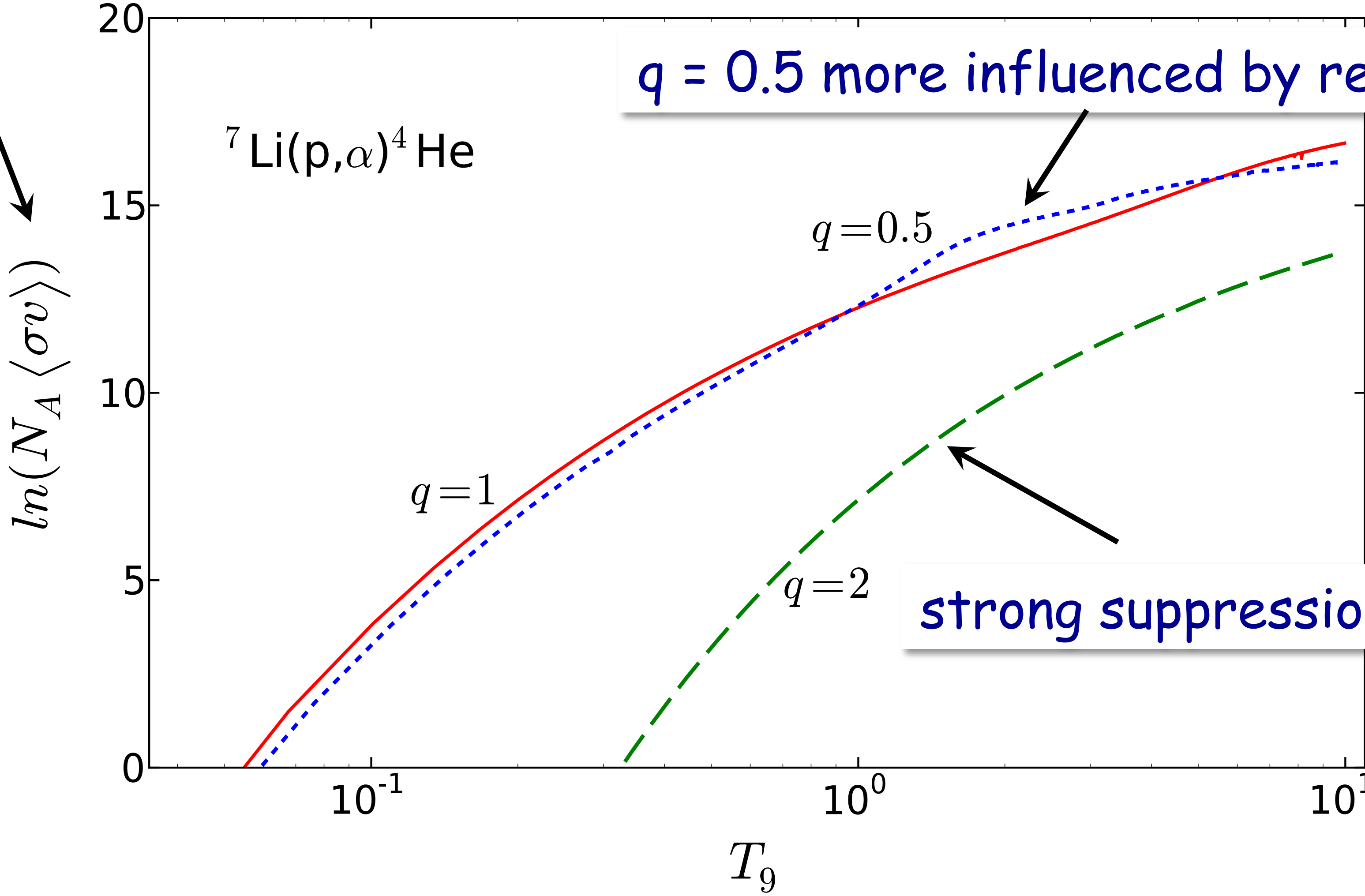
strong suppression for q = 2

Resonant charged reactions: S-factor



Resonant charged reactions: reaction rate

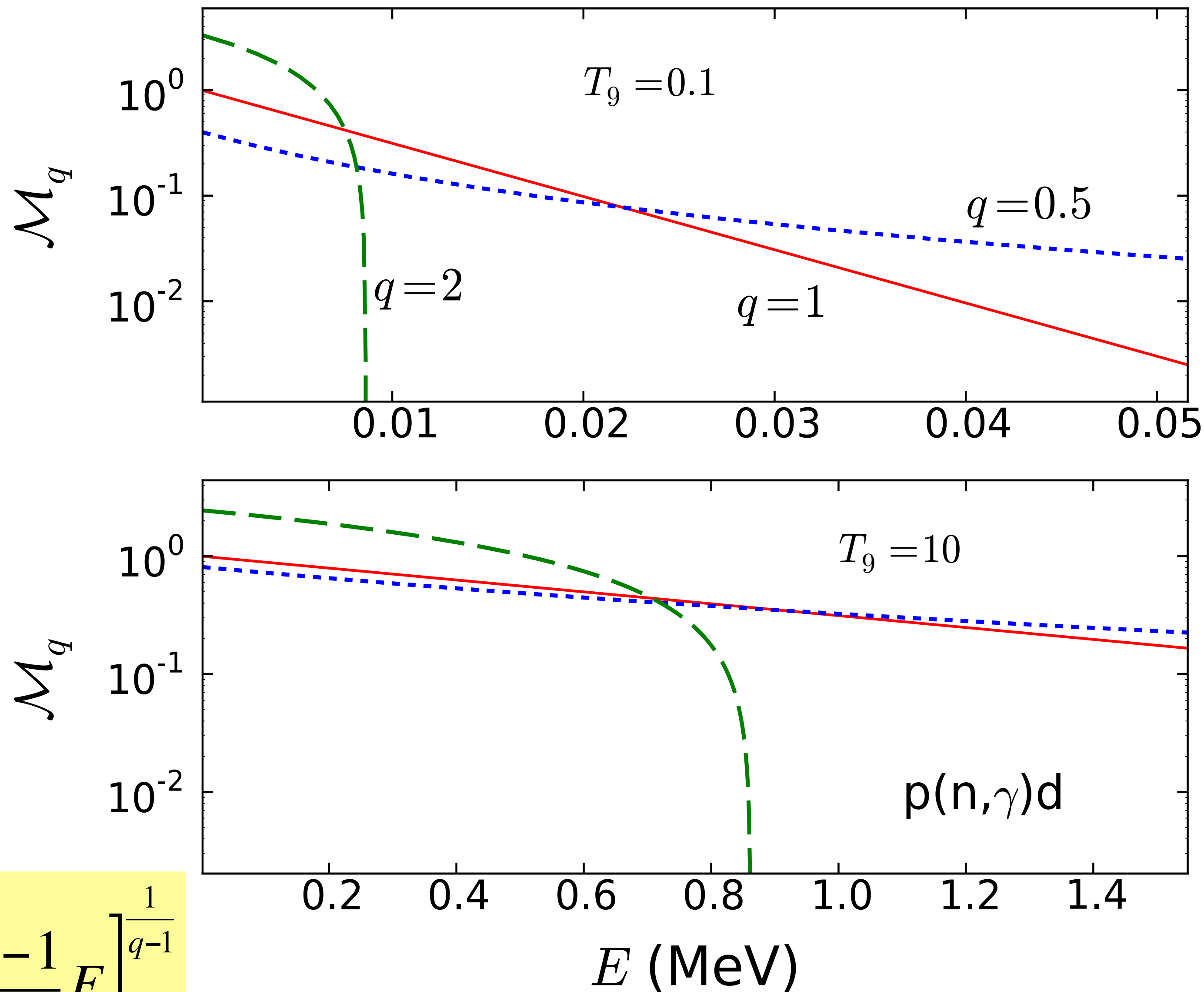
$r_{ij}\ (cm^3/mol/s)$



q = 0.5 more influenced by resonances

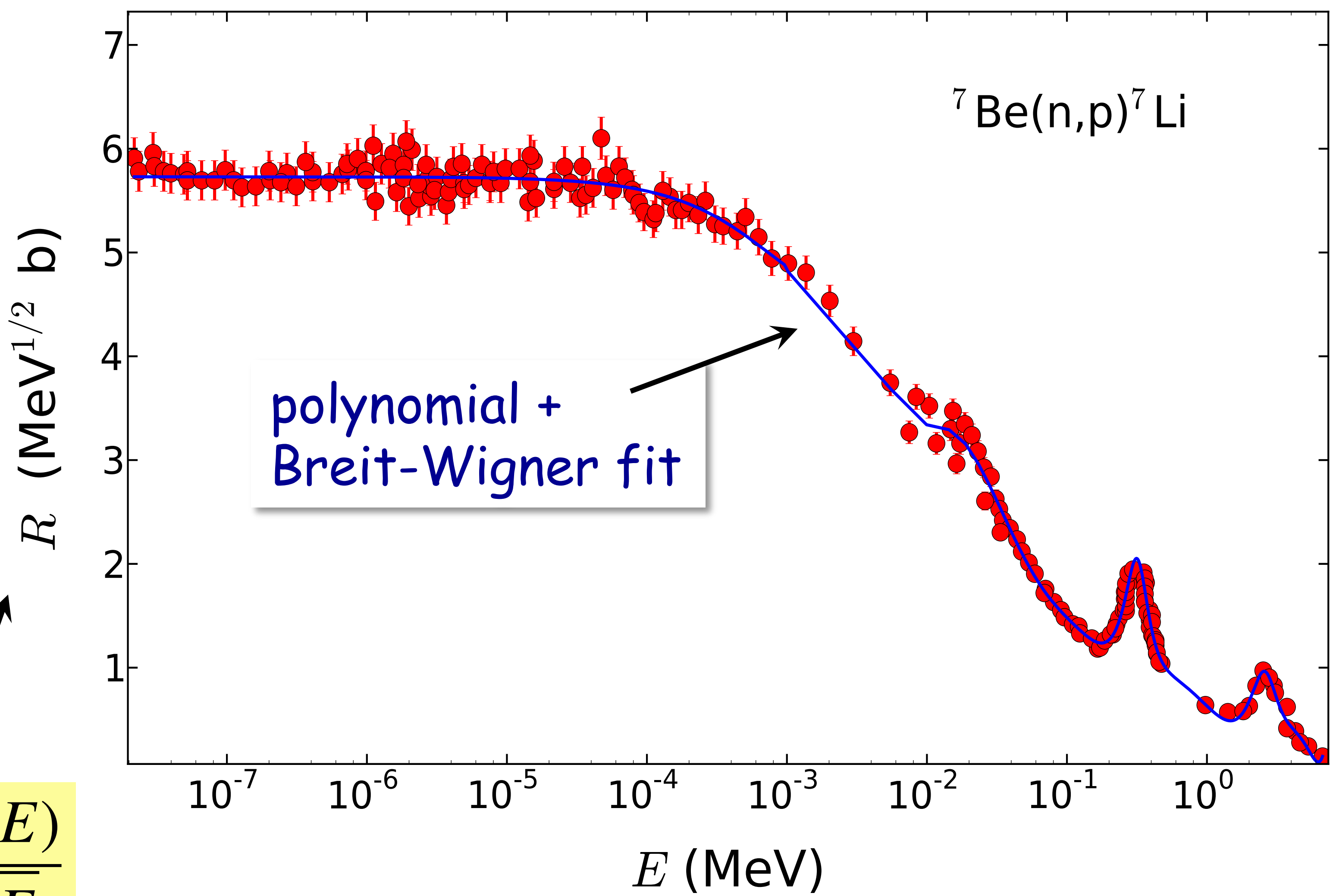
strong suppression for q = 2

Neutron-induced reactions



$$M_q(E,T) = N(q,T)\left[1 - \frac{q-1}{k_B T} E\right]^{\frac{1}{q-1}}$$

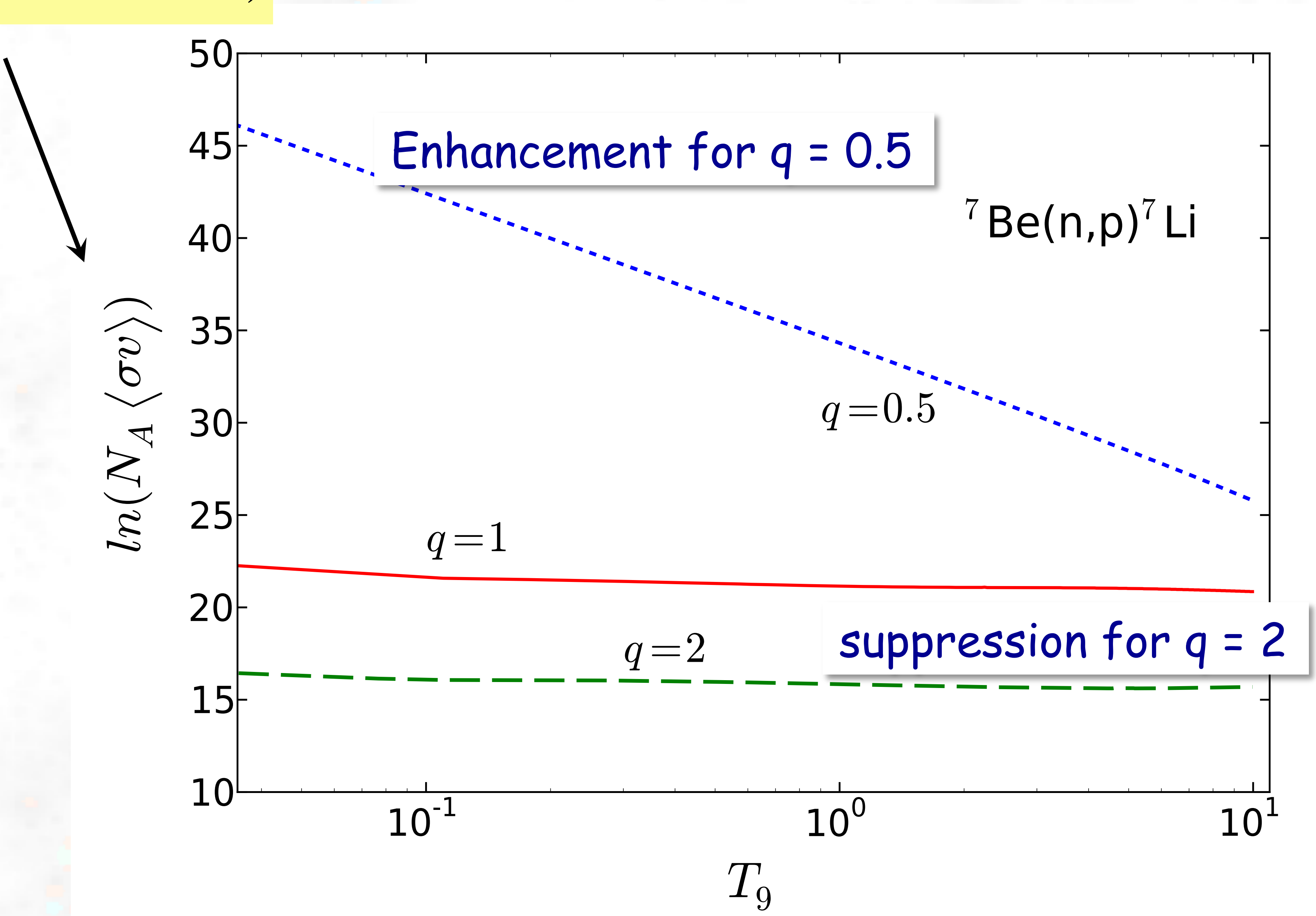
Neutron-induced reactions



$$\sigma = \frac{R(E)}{\sqrt{E}}$$

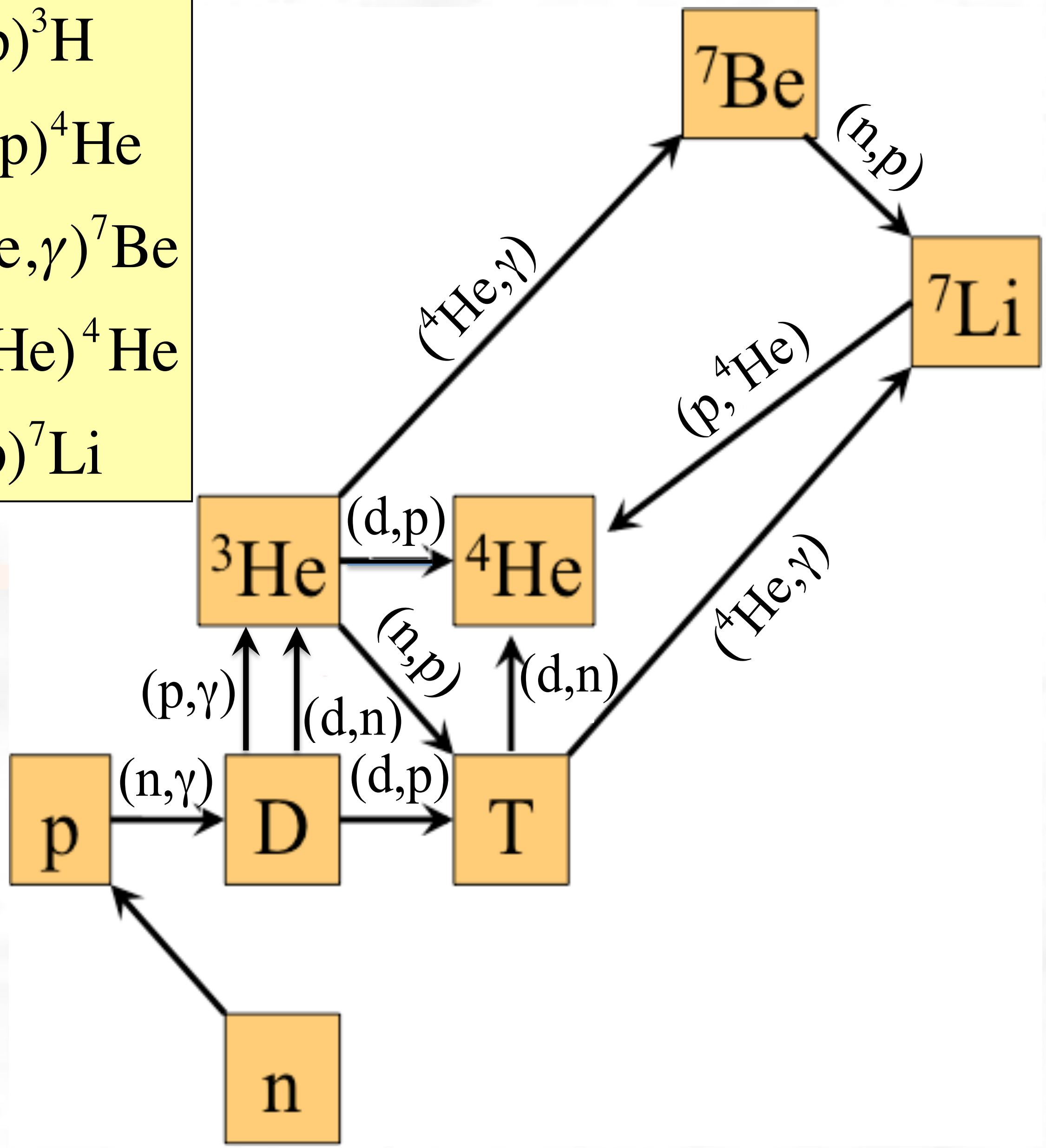
Neutron-induced reactions

$r_{ij}\ (cm^3/mol/s)$



BBN reaction network

1: $n \rightarrow p$	7: $^4He(^3H,\gamma)^7Li$
2: $n(p,\gamma)d$	8: $^3He(n,p)^3H$
3: $d(p,\gamma)^3He$	9: $^3He(d,p)^4He$
4: $d(d,n)^3He$	10: $^4He(^3He,\gamma)^7Be$
5: $d(d,p)^3H$	11: $^7Li(p,^4He)^4He$
6: $^3H(d,n)^4He$	12: $^7Be(n,p)^7Li$

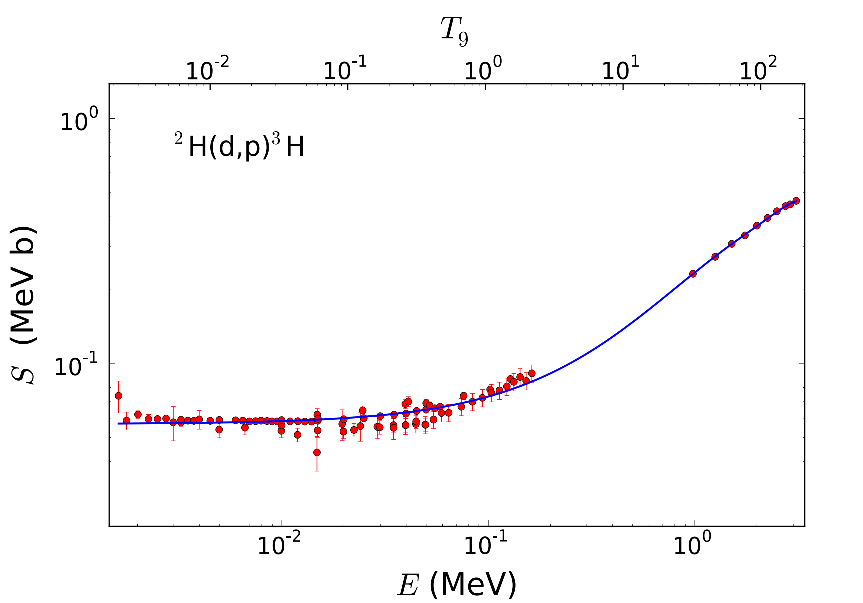


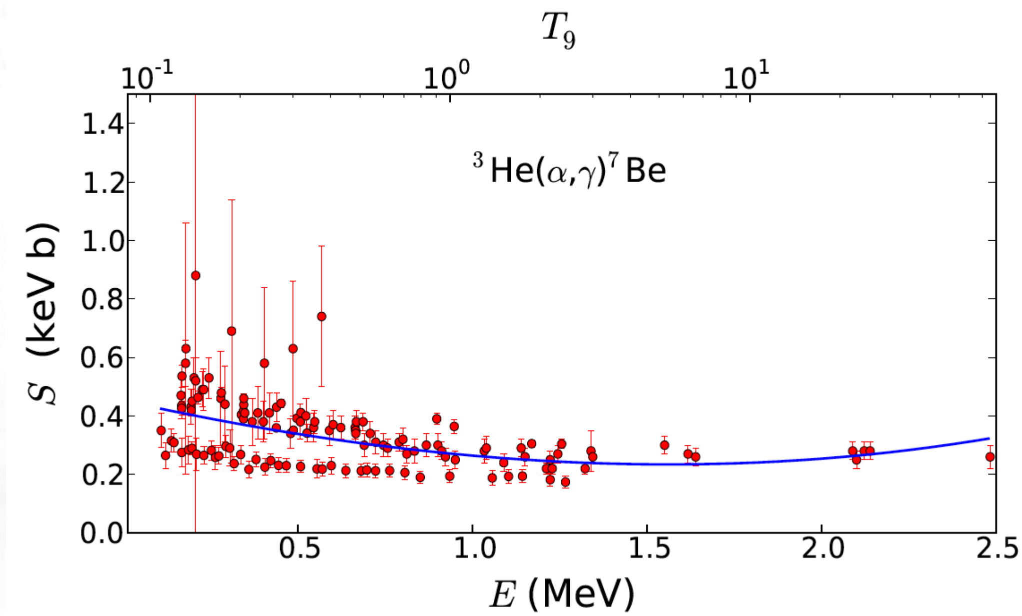
Only considered the 12 main BBN reactions.
Reaction rates re-fitted.

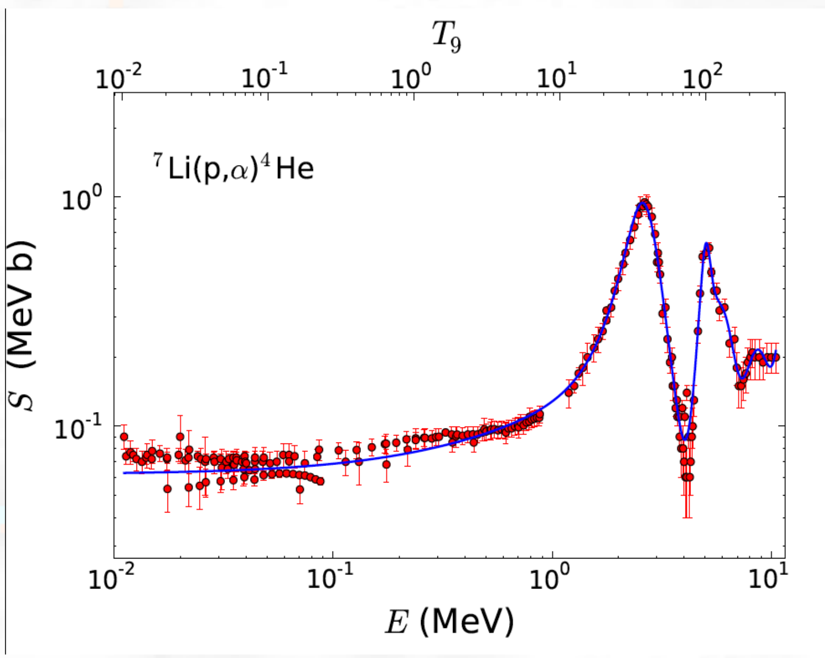
Other reaction ratess from

Smith, Kawano, Malaney, Ap. J. Suppl. 85 (1993) 219.

Data Survey







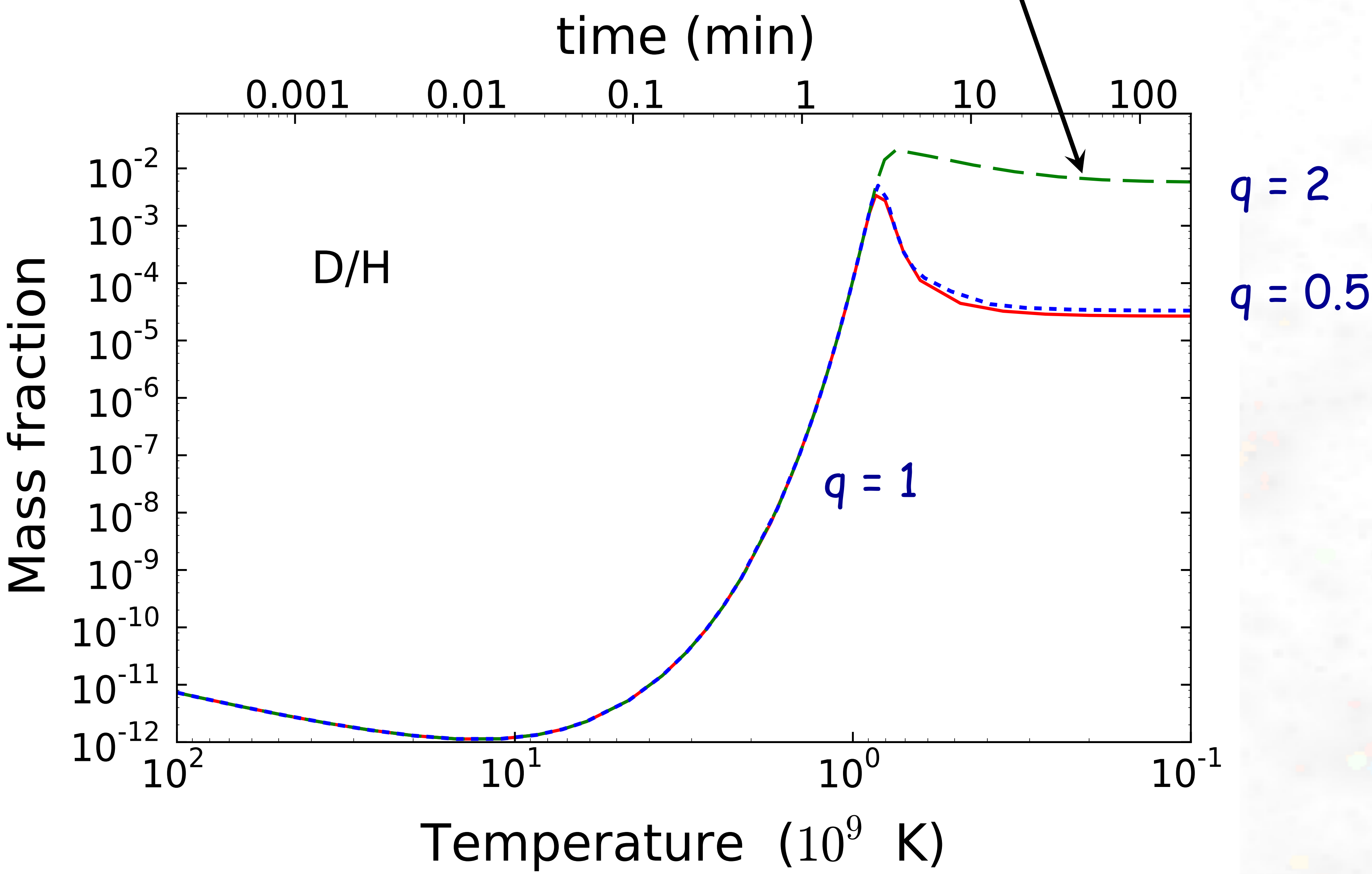
etc.

→ S-factor fit
(function fit: no attempt to reproduce physics, e.g. with R-matrix)

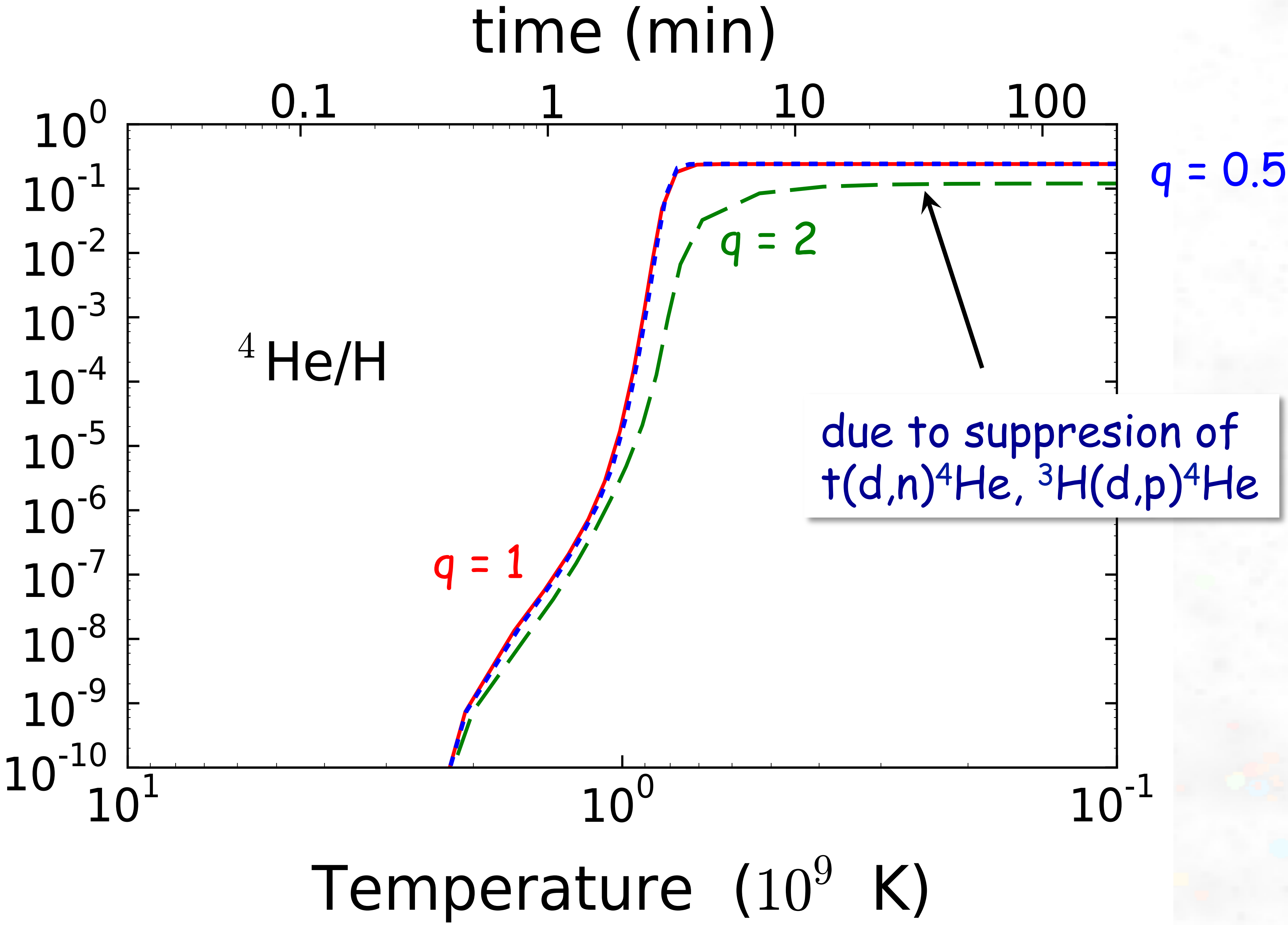
→ rates for extensive & non-extensive statistics

deuterium abundance

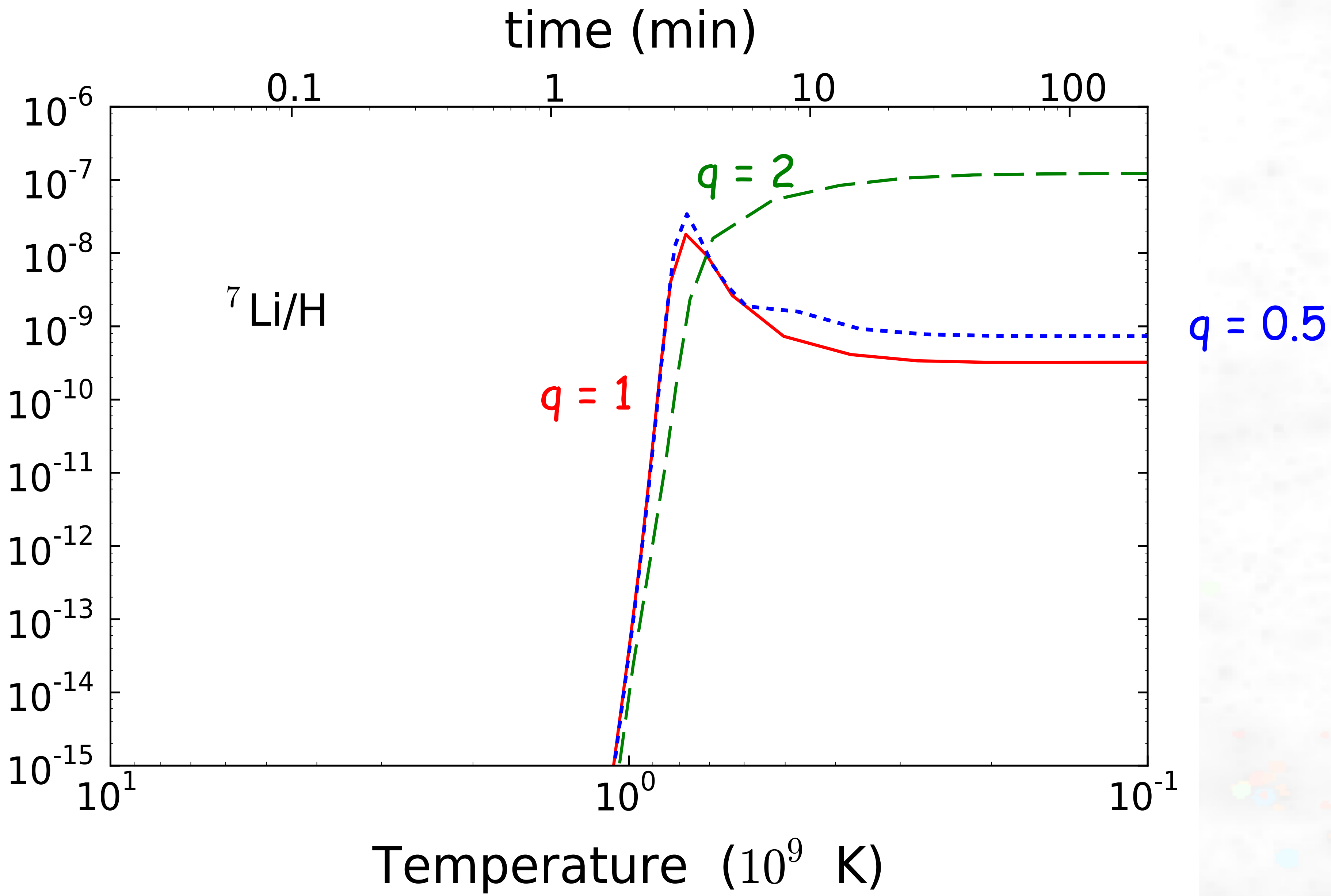
due to suppression of D-destruction



^{4}He abundance



^7Li abundance



Comparison to observations

	BNN	Non-ext. q=0.5	Non-ext. q=2	Observation
^{4}He/H	0.249	0.243	0.141	0.249
D/H	2.62	3.31	570	2.82 x 10^{-5}
^{3}He/H	0.98	0.091	69.1	(0.9 -1.3) x 10^{-5}
^{7}LI/H	4.39	6.89	356.	1.1 x 10^{-10}

non-extensive statistics

standard BBN

Conclusions

- Dark matter with 5 dark sectors compatible with BBN observations. Deserves more investigation.
- Electron screening does not influence BBN. Dead end.
- non-Maxwellian distribution from non-extensive statistics with $q \neq 1 \rightarrow$ our understanding of the cosmic evolution of the universe will have to be significantly changed.
- But fit to BBN observations allow for a possible deviation for the non-extensive parameter

$$q = 1 + 0.05 - 0.1$$

Deserves more investigation.