Outline

- Introduction to core-collapse supernova dynamics
- The neutrino-driven mechanism
- Status of self-consistent models in two dimensions
- The dimension conundrum: How does 3D differ from 2D?
Final Stages of Massive Star Evolution

Stellar Core Collapse and Explosion
Evolved massive star prior to its collapse:

Star develops onion-shell structure in sequence of nuclear burning stages over millions of years

(layers not drawn to scale)
Evolved massive star prior to its collapse:

Star develops onion-shell structure in sequence of nuclear burning stages over millions of years
Gravitational instability of the stellar core:

Stellar iron core begins collapse when it reaches a mass near the critical Chandrasekhar mass limit.

Collapse becomes dynamical because of electron captures and photo-disintegration of Fe-group nuclei.
Core bounce at nuclear density:

Inner core bounces when nuclear matter density is reached and incompressibility increases.

Shock wave forms.
Shock stagnation:

Shock wave loses huge amounts of energy by photo-disintegration of Fe-group nuclei.

Shock stagnates still inside Fe-core

Proto-neutron star
Explosion Mechanism
by
Neutrino Heating
Shock “revival”:

Stalled shock wave must receive energy to start reexpansion against ram pressure of infalling stellar core.

Shock can receive fresh energy from neutrinos!

Proto-neutron star

Shock wave

Si

Accretion

n, p

O
Explosion:

Shock wave expands into outer stellar layers, heats and ejects them.

Creation of radioactive nickel in shock-heated Si-layer.

Proto-neutron star (PNS)
Nucleosynthesis during the explosion:

Shock-heated and neutrino-heated outflows are sites for element formation.
Neutrinos & SN Explosion Mechanism

Paradigm: Explosions by the neutrino-heating mechanism, supported by hydrodynamic instabilities in the postshock layer

- **“Neutrino-heating mechanism”:** Neutrinos `revive' stalled shock by energy deposition (Colgate & White 1966, Wilson 1982, Bethe & Wilson 1985);

Neutrino Heating and Cooling

- Neutrino heating:
  \[ q^+_{\nu} = 1.544 \times 10^{20} \left( \frac{L_{\nu e}}{10^{52} \text{ erg s}^{-1}} \right) \left( \frac{T_{\nu e}}{4 \text{ MeV}} \right)^2 \left( \frac{100 \text{ km}}{r} \right)^2 (Y_n + Y_p) \quad \left[ \frac{\text{erg}}{\text{g s}} \right] \]

- Neutrino cooling:
  \[ C = 1.399 \times 10^{20} \left( \frac{T}{2 \text{ MeV}} \right)^6 (Y_n + Y_p) \quad \left[ \frac{\text{erg}}{\text{g s}} \right] \]

\[ Q^+_{\nu} = q^+_{\nu} M_g \]
\[ \approx 9.4 \times 10^{51} \frac{\text{erg}}{\text{s}} \left( \frac{k_B T_{\nu}}{4 \text{ MeV}} \right)^2 \left( \frac{L_{\nu}}{3 \times 10^{52} \text{ erg/s}} \right) \left( \frac{M_g}{0.01 M_\odot} \right) \left( \frac{R_g}{100 \text{ km}} \right)^{-2} \]

\[ E_N \approx Q^+_{\nu} t_{\text{dwell}} \]
\[ \approx 9.4 \times 10^{50} \text{ erg} \left( \frac{k_B T_{\nu}}{4 \text{ MeV}} \right)^2 \left( \frac{L_{\nu}}{3 \times 10^{52} \text{ erg/s}} \right) \times \left( \frac{M_g}{0.01 M_\odot} \right)^2 \left( \frac{\dot{M}}{0.1 M_\odot \text{s}^{-1}} \right)^{-1} \left( \frac{R_g}{100 \text{ km}} \right)^{-2} \]


Hydrodynamic instabilities
1D-2D Differences in Parametric Explosion Models

- Nordhaus et al. (ApJ 720 (2010) 694) and Murphy & Burrows (2008) performed 1D & 2D simulations with simple neutrino- heating and cooling terms (no neutrino transport but lightbulb) and found up to ~30% improvement in 2D for 15 $M_{\odot}$ progenitor star.

\[ \mathcal{H} = 1.544 \times 10^{20} \left( \frac{L_{\nu_e}}{10^{52} \text{ erg s}^{-1}} \right) \left( \frac{T_{\nu_e}}{4 \text{ MeV}} \right)^2 \times \left( \frac{100 \text{ km}}{r} \right)^2 (Y_n + Y_p) e^{-\tau_{\nu_e}} \left[ \frac{\text{erg}}{\text{g s}} \right] \]

\[ \mathcal{C} = 1.399 \times 10^{20} \left( \frac{T}{2 \text{ MeV}} \right)^6 (Y_n + Y_p) e^{-\tau_{\nu_e}} \left[ \frac{\text{erg}}{\text{g s}} \right] \]
But: Is neutrino heating strong enough to initiate the explosion?

Most sophisticated, self-consistent numerical simulations of the explosion mechanism in 2D and 3D are necessary!
Predictions of Signals from SN Core

- hydrodynamics of stellar plasma
- (nuclear) EoS
- neutrino physics
- Relativistic gravity
- progenitor conditions

SN explosion models

- neutrinos
- LC, spectra
- nucleosynthesis
- gravitational waves
- explosion asymmetries, pulsar kicks
- explosion energies, remnant masses
Explosion Mechanism: Most Sophisticated Current Models
\[
\frac{\partial \sqrt{\gamma} \rho W}{\partial t} + \frac{\partial \sqrt{-g} \rho W \dot{\gamma}^{i}}{\partial x^{i}} = 0, \\
\frac{\partial \sqrt{\gamma} \rho W^{2} v_{j}}{\partial t} + \frac{\partial \sqrt{-g} \left( \rho W^{2} v_{j} \dot{\gamma}^{i} + \dot{\gamma}_{j}^{i} \right)}{\partial x^{i}} = \frac{1}{2} \sqrt{-g} T_{\mu \nu} \frac{\partial g_{\mu \nu}}{\partial x^{i}} + \left( \frac{\partial \sqrt{\gamma} S_{j}}{\partial t} \right)_{c}, \\
\frac{\partial \sqrt{\gamma} \tau}{\partial t} + \frac{\partial \sqrt{-g} \left( \tau \dot{\gamma}^{i} + P \dot{v}^{i} \right)}{\partial x^{i}} = \alpha \sqrt{-g} \left( T_{\mu \nu} \frac{\partial \ln \alpha}{\partial x^{\mu}} - T_{\mu \nu} T^{0}_{\mu \nu} \right) + \left( \frac{\partial \sqrt{\gamma} \tau}{\partial t} \right)_{c}, \\
\frac{\partial \sqrt{\gamma} \rho W Y_{e}}{\partial t} + \frac{\partial \sqrt{-g} \rho W Y_{e} \dot{\gamma}^{i}}{\partial x^{i}} = \left( \frac{\partial \sqrt{\gamma} \rho W Y_{e}}{\partial t} \right)_{c}, \\
\frac{\partial \sqrt{\gamma} \rho W X_{k}}{\partial t} + \frac{\partial \sqrt{-g} \rho W X_{k} \dot{\gamma}^{i}}{\partial x^{i}} = 0.
\]

\[
\Delta \Phi = -2\pi \phi^{5} \left( E + \frac{K_{ij} K^{ij}}{16\pi} \right), \\
\Delta (\alpha \Phi) = 2\pi \alpha \phi^{5} \left( E + 2S + \frac{7K_{ij} K^{ij}}{16\pi} \right), \\
\hat{\Delta} \beta^{i} = 16\pi \alpha \phi^{4} S^{i} + 2\alpha^{10} K^{ij} \hat{\nabla}_{j} \left( \frac{\alpha}{\phi^{6}} \right) - \frac{1}{3} \hat{\nabla}^{i} \hat{\nabla}_{j} \beta^{j}.
\]

CFC metric equations

\[
\frac{\partial W (j + v_{r} \hat{H})}{\partial t} + \frac{\partial}{\partial r} \left[ \left( W \frac{\alpha}{\phi^{2}} - \beta_{r} v_{r} \right) \hat{H} + \left( W v_{r} \frac{\alpha}{\phi^{2}} - \beta_{r} \right) j \right] - \\
\frac{\partial}{\partial r} \left\{ W \varepsilon J \left[ \frac{1}{r} \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) + 2 \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) \frac{\partial \ln \phi}{\partial r} - 2 \frac{\partial \ln \phi}{\partial t} \right] + \\
W \varepsilon \hat{K} \left[ \beta_{r} W \frac{\alpha}{\phi^{2}} - \beta_{r} W + W v_{r} \frac{\alpha}{\phi^{2}} \frac{\partial \ln \phi}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] - \\
W \hat{J} \left[ \frac{1}{r} \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) + 2 \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) \frac{\partial \ln \phi}{\partial r} - 2 \frac{\partial \ln \phi}{\partial t} \right] - \\
W \hat{H} \left[ v_{r} \left( \frac{\beta_{r} - \alpha v_{r}}{\phi^{2}} - 2 \frac{\partial \ln \phi}{\partial r} \right) - \frac{\partial \ln \alpha W}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] + \\
\hat{K} \left[ \frac{\beta_{r} W}{r} - \beta_{r} \frac{\partial W}{\partial r} + W v_{r} \frac{\alpha}{\phi^{2}} \frac{\partial \ln \phi}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] = \alpha \hat{C}^{(0)},
\]

Neutrino transport (VERTEX)

\[
\frac{\partial W (H + v_{r} \hat{K})}{\partial t} + \frac{\partial}{\partial r} \left[ \left( W \frac{\alpha}{\phi^{2}} - \beta_{r} v_{r} \right) \hat{K} + \left( W v_{r} \frac{\alpha}{\phi^{2}} - \beta_{r} \right) \hat{H} \right] - \\
\frac{\partial}{\partial r} \left\{ W \varepsilon \hat{J} \left[ \frac{1}{r} \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) + 2 \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) \frac{\partial \ln \phi}{\partial r} - 2 \frac{\partial \ln \phi}{\partial t} \right] + \\
W \varepsilon \hat{K} \left[ \beta_{r} W \frac{\alpha}{\phi^{2}} - \beta_{r} W + W v_{r} \frac{\alpha}{\phi^{2}} \frac{\partial \ln \phi}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] - \\
W \hat{J} \left[ \frac{1}{r} \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) + 2 \left( \beta_{r} - \frac{\alpha v_{r}}{\phi^{2}} \right) \frac{\partial \ln \phi}{\partial r} - 2 \frac{\partial \ln \phi}{\partial t} \right] - \\
W \hat{H} \left[ v_{r} \left( \frac{\beta_{r} - \alpha v_{r}}{\phi^{2}} - 2 \frac{\partial \ln \phi}{\partial r} \right) - \frac{\partial \ln \alpha W}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] + \\
\hat{K} \left[ \frac{\beta_{r} W}{r} - \beta_{r} \frac{\partial W}{\partial r} + W v_{r} \frac{\alpha}{\phi^{2}} \frac{\partial \ln \phi}{\partial r} + a W^{2} \left( \beta_{r} v_{r} - \frac{\partial v_{r}}{\partial r} \right) \right] = \alpha \hat{C}^{(1)}.
\]
Neutrino Reactions in Supernovae

**Beta processes:**
- $e^- + p \rightleftharpoons n + \nu_e$
- $e^+ + n \rightleftharpoons p + \bar{\nu}_e$
- $e^- + A \rightleftharpoons \nu_e + A^*$

**Neutrino scattering:**
- $\nu + n, p \rightleftharpoons \nu + n, p$
- $\nu + A \rightleftharpoons \nu + A$
- $\nu + e^\pm \rightleftharpoons \nu + e^\pm$

**Thermal pair processes:**
- $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
- $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$

**Neutrino-neutrino reactions:**
- $\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$
  ($\nu_x = \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \text{ or } \bar{\nu}_\tau$)
- $\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$
The Curse and Challenge of the Dimensions

Boltzmann equation determines neutrino distribution function in 6D phase space and time

\[ f(r, \theta, \phi, \Theta, \Phi, \epsilon, t) \]

Integration over 3D momentum space yields source terms for hydrodynamics

\[ Q(r, \theta, \phi, t), \dot{Y}_e(r, \theta, \phi, t) \]

Solution approach

- **3D** hydro + **6D** direct discretization of Boltzmann Eq. (code development by Sumiyoshi & Yamada '12)
- **3D** hydro + two-moment closure of Boltzmann Eq. (next feasible step to full 3D; cf. Kuroda et al. 2012)
- **3D** hydro + "ray-by-ray-plus" variable Eddington factor method (method used at MPA/Garching)
- **2D** hydro + "ray-by-ray-plus" variable Eddington factor method (method used at MPA/Garching)

Required resources

- \( \geq 10–100 \) PFlops/s (sustained!)
- \( \geq 1–10 \) Pflops/s, TBytes
- \( \geq 0.1–1 \) PFlops/s, Tbytes
- \( \geq 0.1–1 \) Tflops/s, < 1 TByte
"Ray-by-Ray" Approximation for Neutrino Transport in 2D and 3D Geometry

Solve large number of spherical transport problems on radial “rays” associated with angular zones of polar coordinate grid

Suggests efficient parallization over the “rays”
Performance and Portability of our Supernova Code *Prometheus-Vertex*

- Code employs **hybrid** MPI/OpenMP programming model (collaborative development with **Katharina Benkert**, HLRS).
- Code has been **ported** to different computer platforms by **Andreas Marek**, High Level Application Support, Rechenzentrum Garching (RZG).
- Code shows **excellent parallel efficiency**, which will be fully exploited in 3D.

**Strong Scaling**

![Strong Scaling Graph](image)
Relativistic 2D CCSN Explosion Models

Color coded: entropy


Basic confirmation of previous explosion models for 11.2 and 15 \( M_{\text{Sun}} \) stars by Marek & THJ (2009)
Relativistic 2D CCSN Explosion Models

"Diagnostic energy" of explosion

Maximum shock radius
2D SN Explosion Models

- Basic confirmation of the neutrino-driven mechanism
- Confirm reduction of the critical neutrino luminosity that enables an explosion in self-consistent 2D treatments compared to 1D
Nucleosynthesis in Neutrino-Heated SN Ejecta

Crucial parameters for nucleosynthesis in neutrino-driven outflows:

* Electron-to-baryon ratio $Y_e$ (----> neutron excess)
* Entropy (----> ratio of (temperature)$^3$ to density)
* Expansion timescale

Determined by the interaction of stellar gas with neutrinos from nascent neutron star:

\[ \nu_e + n \rightarrow e^- + p \]
\[ \bar{\nu}_e + p \rightarrow e^+ + n \]

\[ Y_e \sim \left[ 1 + \frac{L_{\bar{\nu}_e}(e_{\bar{\nu}_e} - 2\Delta)}{L_{\nu_e}(e_{\nu_e} + 2\Delta)} \right]^{-1} \]

with $\epsilon_\nu = \frac{\langle \epsilon^2_\nu \rangle}{\langle \epsilon_\nu \rangle}$ and $\Delta = (m_n - m_p)c^2 \approx 1.29$ MeV.

If $L_{\bar{\nu}_e} \approx L_{\nu_e}$, one needs for $Y_e < 0.5$ (i.e. neutron excess):

\[ \epsilon_{\bar{\nu}_e} - \epsilon_{\nu_e} > 4\Delta. \]
Nucleosynthesis in Neutrino-Heated SN Ejecta

Convectively ejected n-rich matter makes ONeMg-core and low-mass Fe-core supernovae an interesting source of nuclei between the iron group and \( N = 50 \) (from Zn to Zr), possibly also of weak r-process nuclei.

Support for 2D CCSN Explosion Models

AXISYMMETRIC AB INITIO CORE-Collapse SUPERNOVA SIMULATIONS OF 12–25 $M_\odot$ STARS

Stephen W. Bruenn\textsuperscript{1}, Anthony Mezzacappa\textsuperscript{2,3,4}, W. Raphael Hix\textsuperscript{2,3}, Eric J. Lentz\textsuperscript{3,2,5}, O. E. Bronson Messer\textsuperscript{6,3,4}, Eric J. Lingerfelt\textsuperscript{2,4}, John M. Blondin\textsuperscript{7}, Eirik Endeve\textsuperscript{8}, Pedro Marronetti\textsuperscript{1,8}, and Konstantin N. Yakunin\textsuperscript{1}

2D explosions for 12, 15, 20, 25 $M_\odot$ progenitors of Woosley & Heger (2007)

Bruenn et al., arXiv:1212.1747
2D SN Explosion Models

Results and numerical approaches of different groups still differ in many aspects:

- Different explosion behavior and different explosion energies
- Different codes, neutrino transport schemes and reactions, EoS treatment

Direct comparisons are urgently needed!
2D explosions seem to be “marginal”, at least for some progenitor models and in some (the most?) sophisticated simulations.

Nature is three dimensional, but 2D models impose the constraint of axisymmetry.

Turbulent cascade in 3D transports energy from large to small scales, which is opposite to 2D.

Is 3D turbulence more supportive to an explosion? Is the third dimension the key to the neutrino mechanism?

3D models are needed to confirm explosion mechanism suggested by 2D simulations!
3D vs. 2D Differences:
The Dimension Conundrum
2D-3D Differences in Parametric Explosion Models


\[
\mathcal{H} = 1.544 \times 10^{20} \left( \frac{L_{\nu e}}{10^{52} \text{ erg s}^{-1}} \right) \left( \frac{T_{\nu e}}{4 \text{ MeV}} \right)^2 \\
\times \left( \frac{100 \text{ km}}{r} \right)^2 \left( Y_n + Y_p \right) e^{-\tau_{\nu e}} \left[ \frac{\text{erg}}{\text{g s}} \right]
\]

\[
C = 1.399 \times 10^{20} \left( \frac{T}{2 \text{ MeV}} \right)^6 \left( Y_n + Y_p \right) e^{-\tau_{\nu e}} \left[ \frac{\text{erg}}{\text{g s}} \right]
\]
2D-3D Differences in Parametric Explosion Models

- F. Hanke (Diploma Thesis, MPA, 2010) in agreement with L. Scheck (PhD Thesis, MPA, 2007) could not confirm the findings by Nordhaus et al. (2010)! 2D and 3D simulations for 11.2 \( M_{\text{sun}} \) and 15 \( M_{\text{sun}} \) progenitors are very similar but results depend on numerical grid resolution: 2D with higher resolution explodes easier, 3D shows opposite trend!


2D & 3D slices for 11.2 \( M_{\text{sun}} \) model, \( L = 1.0 \times 10^{52} \) erg/s
Growing "Diversity" of 3D Results

- Dolence et al. (arXiv:1210.5241) find much smaller 2D/3D difference of critical luminosity, but still slightly earlier explosion in 3D.


- Couch (arXiv:1212.0010) finds also later explosions in 3D than in 2D and higher critical luminosity in 3D! But critical luminosity increases in 2D with better resolution.

- Ott et al. (arXiv:1210.6674) reject relevance of SASI in 3D and conclude that neutrino-driven convection dominates evolution.

Reasons for 2D/3D differences and different results by different groups are not understood!
Growing "Diversity" of 3D Results

- These results do not yield a clear picture of 3D effects.

But:

- The simulations were performed with different grids (cartesian+AMR, polar), different codes (CASTRO, ZEUS, FLASH, Cactus, Prometheus), and different treatments of input physics for EOS and neutrinos, some with simplified, not fully self-consistent set-ups.

- Resolution differences are difficult to assess and are likely to strongly depend on spatial region and coordinate direction.

- Partially compensating effects of opposite influence might be responsible for the seemingly conflicting results.

- Convergence tests with much higher resolution and detailed code comparisons for “clean”, well defined problems are urgently needed, but both will be ambitious!
Full-Scale 3D Core-Collapse Supernova Models with Detailed Neutrino Transport
3D Supernova Models

PRACE grant of 146.7 million core hours allows us to do the first 3D simulations on 16,000 cores.
Computing Requirements for 2D & 3D Supernova Modeling

Time-dependent simulations: \( t \sim 1 \text{ second}, \sim 10^6 \text{ time steps}! \)

CPU-time requirements for one model run:

🌟 In 2D with 600 radial zones, 1 degree lateral resolution:

\(~ 3 \times 10^{18} \text{ Flops, need } \sim 10^6 \text{ processor-core hours.} \)

🌟 In 3D with 600 radial zones, 1.5 degrees angular resolution:

\(~ 3 \times 10^{20} \text{ Flops, need } \sim 10^8 \text{ processor-core hours.} \)
3D Core-Collapse Models

11.2 $M_{\text{Sun}}$ progenitor

(Janka et al., PTEP 2012)

Florian Hanke, PhD project
3D Core-Collapse Models

27 \( M_{\text{Sun}} \) progenitor

Florian Hanke, PhD project
Laboratory Astrophysics

"SWASI" Instability as an analogue of SASI in the supernova core
Foglizzo et al., PRL 108 (2012) 051103

Constraint of experiment: No convective activity
Numerical Convergence?

2D simulations are converged; no difference between 0.7, 1.4, and 2.04 degrees angular resolution.

But: 3D simulations may need more resolution for convergence than in 2D!
Numerical Convergence?


Turbulent energy cascade in 2D from small to large scales, in 3D from large to small scales! ➔ More than 2 degree resolution needed in 3D!

Figure 16. Turbulent energy spectra \( E(l) \) as functions of the multipole order \( l \) for different angular resolution. The spectra are based on a decomposition of the azimuthal velocity \( v_\theta \) into spherical harmonics at radius \( r = 150 \text{ km} \) and 400 ms post-bounce time for 15 \( M_\odot \) runs with an electron–neutrino luminosity of \( L_{\nu_e} = 2.2 \times 10^{52} \text{ erg s}^{-1} \). Left: 2D models with different angular resolution (black, different thickness) and, for comparison, the 3D model with the highest employed angular resolution (gray). Right: 3D models with different angular resolution and, for comparison, the 2D model with the highest employed angular resolution (gray). The power-law dependence and direction of the energy and enstrophy cascades (see the text) are indicated by red lines and labels for 2D models in the left panel and 3D models in the right panel. The left vertical, dotted line roughly marks the energy-injection scale, and the right vertical, dotted line denotes the onset of dissipation at high \( l \) for the best-displayed resolution.
Summary

- Modelling of SN explosion mechanism has made considerable progress.
- 2D relativistic models yield explosions for “soft” EoSs. Explosion energy tends to be on low side (except recent models by Bruenn et al., arXiv:1212.1747).
- 3D modeling has only begun. No clear picture of 3D effects yet. But SASI can dominate (certain phases) also in 3D models!
- 3D SN modeling is extremely challenging and variety of approaches for neutrino transport and hydrodynamics/grid choices will be and need to be used.
- Numerical effects (and artifacts) and resolution dependencies in 2D and 3D models must still be understood.
- Bigger computations on faster computers are indispensable, but high complexity of highly-coupled multi-component problem will demand special care and quality control.
For concise reviews of most of what I will say, see

and
PTEP 2012, 01A309, arXiv:1211.1378

Explosion Mechanisms of Core-Collapse Supernovae

Hans-Thomas Janka

Max Planck Institute for Astrophysics, D-85748 Garching, Germany;
email: thj@mpa-garching.mpg.de